

CHAPTER 2

Logical matters

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CHAPTER 2

Logical matters

2.1 Introduction

This chapter introduces a set of fundamental conceptual tools, mostly drawn from the field of logic, which, because of their wide currency in discussions of semantic matters, constitute indispensable background knowledge for a study of meaning in language. The level of treatment here is fairly elementary; some of the notions introduced will be further refined in subsequent chapters.

2.2 Arguments and predicates

A closely linked pair of concepts which are absolutely fundamental to both logic and semantics are **argument** and **predicate**. No attempt will be made here to explore the philosophical background and underpinning of these notions: the basic notions are fairly accessible and they will be treated in an elementary fashion. Put simply, an argument designates some entity or group of entities, whereas a predicate attributes some property to the entity denoted by the argument, or a relation between the entities denoted by the arguments, if there is more than one. Thus, in *John is tall*, we can identify *John* as the argument, and *is tall* as the predicate. In *John likes Mary*, both *John* and *Mary* are arguments, and *likes* is the predicate which attributes a particular relationship between the entities denoted by the arguments; in *John gave Mary a rose*, there are three arguments, *John*, *Mary* and *the rose*, with *gave* as the predicate. The combination of an argument and a predicate forms a proposition: notice that a proposition may have only one predicate, but may have more than one argument. It is not clear whether there is any theoretical upper limit to the number of arguments a predicate may take, but the most one is likely to encounter in linguistic semantic discussions is four, exemplified by *Mary paid John £500 for the car*.

Arguments: *Mary, John, £500, the car*

Predicate: *paid (for)*

An argument may have a more or less complex internal structure. For instance, a whole proposition, itself possessing argument(s) and predicate, may constitute an argument, as in *John was surprised that the man was tall*

Arguments: (i) *John (li) that the man was tall*

Predicate: *was surprised*

Argument (ii) Argument: *the man*

Predicate: *was tall*

There are various ways of incorporating propositions as constituents of complex arguments, and there is no limit to the resulting degree of complexity. An account of this is beyond the scope of this book.

Predicates are commonly described as **one-place**, **two-place**, **three-place**, etc. according to the number of arguments they take, so that, for instance, *is poor* is a one-place predicate, and *teach* is a three-place predicate (*John taught Mary French*'), But what does it mean to say that *teach* is a three-place predicate? How do we determine how many places a predicate has?

This is, in fact, a very difficult question, but we can get some handle on it by looking at a few verbs. Let us start by looking at *teach*. One aspect of the problem is immediately obvious when we look at examples such as the following:

- (1) John taught Mary French.
- (2) John taught French for two years.
- (3) A: What does John do?
B: I think he teaches at Lowhampton High.
- (4) Anybody who teaches teenagers should get double salary.

Do we say that *teach* is three-place in (1), two-place in (2) and (4), and one-place in (3)? While there is some justification for such an analysis, there is intuitively a sense in which the (overt) argument structure of (1) is basic, and irreducible. Logically, for an act of teaching to take place, there must be someone who does the teaching, someone who undergoes the teaching (whether or not they actually learn anything!), and some item of knowledge or skill which it is hoped will be acquired by the latter. Without at least one each of these requirements, the notion of teaching is not logically coherent. What, then, are we to make of (2)-(4) above? It seems that we assume that the missing arguments could in principle be supplied, but the speaker has not supplied them, presumably because they are not currently relevant, or perhaps in some cases because they are extremely obvious. In (2), we do not have a vision of John discoursing in solitude on the French language; nor do we imagine that John in (3) does something like sneezing, which needs neither audience nor topic. By the same sort of criteria, *read* is a basically two-place verb, in spite of the existence of sentences such as *John is reading* and *John is learning to read*. We cannot make sense of *read* unless we have someone to do the reading and some coded signal to decode. As a final example, consider *buy*. This requires four

arguments, as in *John sold the car to Mary for £500*. If John receives no money, he is simply giving the car away; if there is no car, Mary is just giving John some money; if there is no one to receive the money and concede ownership of the car, then Mary is throwing her money away and taking possession of the car. And so on. In this way, it is usually possible to determine a basic logically minimum number of arguments for a predicate.

But our problems are not over. Acts of reading, teaching, and selling (and sneezing) take place at particular times and particular places. We may say that unlocated, timeless acts of reading, etc., are logically impossible. It is also necessarily the case that the ambient temperature has such and such a value. Whereas it is relatively easy to establish the *minimum* number of arguments for a predicate, how do we establish a *maximum*? Or perhaps there is no such thing? After all, if we learn that John sneezed, we assume that the event happened at a particular place and time, and that even if the speaker did not specify these items, they could in principle be specified, along with countless other things. What is the difference in status, therefore, between the time and place of John's sneezing, and the subject that John teaches? One approach is to say that although a verb like *sneeze* in a sense conjures up a rather complex picture of a person in a setting acting in a certain way, it highlights only certain aspects of that scene, but without obliterating or denying the rest. These highlighted aspects are what distinguish the act denoted from other possible acts. This means that if we wish to ascertain whether a John-sneeze-event has occurred, we need only observe happenings pertaining closely to John; we do not need to check time, place, or temperature. In this way we can check that the minima we previously established for the number of arguments a predicate takes are generally also maxima.

2.3 Sense, denotation, and reference: *intension and extension*

Language is used to communicate about things, happenings, and states of affairs in the world, and one way of approaching the study of meaning is to attempt to correlate expressions in language with aspects of the world. This is known as the **extensional** approach to meaning.

The thing or things in the world referred to by a particular expression is its **referent(s)**: in saying *The cat's hungry*, I am (normally) referring to a particular cat, and that cat is the referent of the expression *the cat*. The whole utterance attributes a particular state to the cat in question. We can also consider the whole class of potential referents of the word *cat*, namely, the class of cats. This, too, is sometimes called the **reference** of the word *cat*. But this is clearly different from the designation of particular individuals as in the case of *The cat's hungry*, so, to avoid confusion, we shall follow Lyons and say that the class of cats constitutes the **denotation** of the word *cat*. So, in the case of

The cat's hungry, the word *cat* denotes the class of cats, but *the cat* refers to a particular cat.

The alternative to an extensional approach to meaning is an **intensional** approach. Take the word *cat*. Why do we use it to refer to cats, rather than, say, to platypuses or aardvarks or spiny anteaters? One answer is that the word is associated with some kind of mental representation of the type of thing that it can be used to refer to, and aardvarks do not fit the description associated with the word *cat*. This representation constitutes what is called the **sense** of the word (or at least part of it). We shall assume in this book that the main function of linguistic expressions is to mobilize concepts, that concepts are the main constituents of sense, and that sense (and hence concepts) constrains (even if it does not completely determine) reference. (It should be noted that some authors, for instance Lyons, understand *sense* in a different way. For them, sense is a matter of the relations between a word and other words in a language. So, for instance, the sense of *cat* would be constituted by its relations with other words such as *dog* (a cat is necessarily not a dog), *animal* (a cat is an animal), *miaow* (*The cat miaowed* is normal but *?The dog miaowed* is not.)

2.4 Sentence, statement, utterance and proposition

A number of distinctions need now to be made which at first sight might seem to be academic nit-picking of the worst sort. However, they are absolutely vital for clarity in semantics. For the sake of simplicity of exposition, the distinctions will first of all be explained in connection with declarative sentences only; how the various notions apply to non-declarative sentences will be dealt with later.

2.4.1 Sentence meaning; truth conditions; propositional content

A sentence is a grammatical unit, that is, it is a string of words of a particular type, whose well-formedness conditions are specified in the grammar of the language. Thus, *The cat sat on the mat* and *John put his hat on the table* are sentences of English; *John put on the table* and *Is the of mother boy swim* are not. We shall not spend too much time on discussing what distinguishes a sentence from other grammatical units, such as a phrase, or a word, except to say that a sentence must contain at least one independent clause (i.e., one that does not need another clause, to be grammatical), and a clause must contain at least one predication (i.e., an argument-predicate structure). The sentence is thus the smallest linguistic unit that can be used in an act of 'telling'. Compare the following for normality/oddness:

- (5) A: Tell me something nice.
B: Chocolate.

A: What do you mean?

B: Well, chocolate is nice.

(6) A: Tell me something nice.

B: Love is a many-splendoured thing.

A: Ah! How true!

The oddness of B's reply in (5) consists in the fact that it neither overtly forms a sentence, nor permits a plausible sentence to be reconstructed from the context, as in, for instance, (7):

(7) A: He asked me what I wanted.

B: What did you tell him?

A: Icecream.

Here, the full act of telling is reconstructible as *I want ice cream*, which involves a sentence. Henceforward we shall use our intuitions as to what constitutes a sentence.

Notice that we said above that the sentence was 'used in the act of telling': the sentence itself, on its own, does not in fact tell us anything. What does the sentence *The cat sat on the mat* tell us? Is it true? There is no way of knowing, or rather, the question does not make sense: sentences of themselves do not necessarily have truth values (some, arguably, have: *A molecule of water contains two hydrogen atoms and one oxygen atom*). Yet presumably no one would wish to say that *The cat sat on the mat* was meaningless. Let us introduce the notion **sentence meaning** to designate the semantic properties a sentence possesses merely by virtue of being a well-formed sentence (of English), before any question of context or use arises. A sentence possesses this meaning exclusively by virtue of the words it contains, and their grammatical arrangement. (I assume, here, that words have at least some context-independent conventionally assigned semantic properties.) We may assume that the grammar of a language is associated with principles of composition, that is, rules which tell us how to put together the meanings of the constituents of a construction to get the global meaning of the construction. Thus, in *The big cat sat on the small mat* we know, for instance, that smallness is attributed to *the mat* and not to *the cat*, and that a superior vertical position is attributed to *the cat*; we know, furthermore, what sort of animal is involved, and that only one of them would be intended as a referent in any actual use of the sentence. And so on. In a fairly obvious sense, the meaning of a sentence will constrain the uses to which it can be put, at least without the setting up of additional *ad hoc* conventions. (For example, *If I say How are things?, you will know it's my husband on the phone*: that is not what *How are things?*, by general convention, means.)

Although a sentence, outside of particular uses, does not have a truth value, it does have truth conditions, that is, conditions which must hold for the sentence to be used to make a true statement. Thus, before we can truthfully

say, on some occasion, *The cat is on the mat*. there must be some relevant feline occupying a specific position relative to an appropriate item of floor-covering. Those aspects of the meaning of a sentence which determine whether a statement the sentence is being used to make, in a particular situation, is true or false, are collectively known as the **propositional content** of the sentence. Two sentences with identical propositional content will yield statements with the same truth values on all occasions of use, as for instance: *John caressed Mary* and *Mary was caressed by John*. By the same token, if two sentences have different propositional content, there will necessarily exist some conceivable situation in which they will yield statements with opposite truth values.

Propositional content does not by any means make up the whole of sentence meaning, as conceived here. For instance, the interrogative meaning of a question is not included, nor the imperative meaning of a command (more generally, illocutionary meaning falls outside propositional content). The force of such words as *yet*, *still* and *already* (sometimes called **conventional implicature**) is excluded; the following two sentences, for instance, have the same propositional content, since their truth conditions are the same, but one would not want to say that they were identical in meaning (nor are they *appropriate* in the same circumstances):

- (8) John has not arrived.
- (9) John has not yet arrived.

Other aspects of meaning which do not fall under propositional content are expressive meaning:

- (10) It's very cold in here.
- (11) It's bloody cold in here.

and features of register such as the formal/colloquial distinction:

- (12) My old man kicked the bucket yesterday.
- (13) My father passed away yesterday.

To qualify as part of sentence meaning, a semantic property simply has to be a stable conventional property of some linguistic expression. We shall not even demand that it be manifested on every occasion of use of the expression in question: it is sufficient that it be potentially present, and not attributable to context. We shall be particularly impressed by evidence that expressions with closely similar propositional content do not possess the property in question. An example of this is the potential expressive nature of the word *baby* compared with *infant* (see Chapter 3 for a more extensive discussion of non-propositional meaning).

2.4.2 Statement meaning

One does not make a statement simply by producing a sentence of declarative

form. Someone in a language class, for instance, practising the tenses of English, who says:

The cat sat on the mat.

The cat sits on the mat.

The cat will sit on the mat.

is not making a series of statements. The first requirement for a statement to have been made is that a proposition must have been expressed; the second requirement is that an appropriate commitment be made to the truth of the proposition. Let us dwell on this for a moment, beginning with the notion of a **proposition**.

2.4.2.1 Propositions

A simple proposition attributes some property to an entity, or a relation between two or more entities. It is either true or false (even if it is not practically or even physically possible to ascertain which): truth or falsehood is a sign that at least one proposition has been expressed. A proposition is not a specifically linguistic entity (although we shall not dwell too long on the problem of what sort of entity it is, and in what Platonic or other realm it subsists). The same proposition may be expressed by an indefinitely large number of sentences:

John saw Mary.

John saw his sister.

Mary was seen by Peter's uncle.

etc.

These can all express the same proposition, provided, of course, that Mary is John's sister, and that John is Peter's uncle, and so on.

Why, then, is *The cat sat on the mat* not a proposition? As it stands, it is neither true nor false. It becomes true or false when it is asserted of some specific cat and some specific mat. Until definite referring expressions in a sentence have been assigned referents, it does not express a specific proposition. A sentence like *The cat sat on the mat* can be used to express an indefinitely large number of different propositions (i.e., with reference to different cats and different mats) on different occasions of use.

2.4.2.2 Epistemic commitment

A proposition may be 'entertained', without any stance being adopted towards its truth or falsehood, as, for instance, in a logic class, where propositions are entertained, and their interrelationships studied, such as

- (14) All aardvarks are purple.
 Tinkerbelle is an aardvark.
 Hence, Tinkerbelle is purple.

However, a proposition on its own cannot actually communicate anything: it is not an item of knowledge. To communicate, it must be energized with some kind of illocutionary force. For instance, the proposition expressed by *The earth is a polyhedron* is of no interest until someone claims it is true (or false): a true or false proposition is an item of knowledge. The illocutionary force in such a case would be that of assertion. A (minimum) statement is therefore a proposition uttered, as Lyons puts it, ‘with epistemic commitment’. There are other possibilities for illocutionary force, to be studied in due course.

2.4.2«3 Incomplete propositions

Consider the following exchange:

(15) A: What’s the time?

B: Half-past four.

Is B telling A anything? Obviously he is. Then what proposition is he expressing? Again the answer is obvious: “The time is 4.30”. But this is not what he actually *says*. In such cases, it is up to the hearer to reconstruct the full form of the proposition on the basis of contextual clues, but there is no doubt that it is the full form of the proposition that the speaker intends to convey, and this should form part of statement meaning.

We are now in a position to spell out what **statement meaning** consists of. First of all, statement meaning incorporates in its entirety all aspects of sentence meaning which belong to the sentence used in making the statement. Secondly, statement meaning includes a specific proposition or propositions, the identification of which requires that (a) referents be assigned to any definite referring expressions in the sentence, and (b) incomplete propositions be completed. It is assumed that the proposition(s) is expressed with the appropriate epistemic commitment.

2.4.3 Utterance meaning

Even a fully elaborated statement meaning may not adequately represent everything the speaker intended to convey by uttering the sentence to which it applies. To take a very simple example, consider the following:

(16) A: Have you cleared the table and washed the dishes?

B: I’ve cleared the table.

In normal circumstances, it would clearly be part of B’s intended message that s/he had not washed the dishes. Yet this cannot be obtained by elaborating or completing the proposition expressed. This is even clearer in the following case where it is obviously the speaker’s intention to convey the proposition that A is too late for supper:

(17) A: Am I in time for supper?

B: I’ve cleared the table.

These extra (i.e. covert) propositions are expected to be inferred by the hearer on the basis of contextual information, but they go well beyond the mere filling out of missing bits in what is actually said. We shall give the name **utterance meaning** to the totality of what the speaker intends to convey by making an utterance, within certain necessary limits.

Two people might, for instance, have an arrangement such that *How was your day?* means "How was your day?", but *How was the day today?* means "My husband is going to his club this evening—we'll have plenty of time for fun". We would wish to exclude this sort of meaning from utterance meaning. I think the key point is that such a use requires a special *ad hoc* stipulation on the part of speaker and hearer: the hearer's understanding of the clue does not arise entirely from his general knowledge of the rules and conventions governing the use of the forms in question.

Once again, utterance meaning subsumes statement meaning, but only in the sense that the latter must be traversed in order to arrive at the former, i.e. is a necessary step in derivation: utterance meaning does not necessarily incorporate statement meaning as a proper part (although, of course, it may do).

2.4.4 Non-declaratives

So far, we have looked only at declarative sentences. However, the notions elaborated above apply equally to non-declarative sentences. Take the case of a question. The general notion of sentence meaning is probably unproblematic here. The equivalent to statement meaning will be **question meaning**. Recall that the propositional content of a statement determined whether it was true or false in a given situation. Now questions do not have truth values, but they do have propositional content. Although a question does not have a truth value, a given question-statement pair has what might be called an **answer value**, that is, the statement is or is not an answer to the question, and if it is, it is either true or false. This answer value is determined equally by the propositional content of the question and the statement:

- (18) A: What day is it today? (said on Tuesday)
 B: It's Monday today, (false answer)
 A: What day is it today? (said on Monday)
 B: It's Monday today, (true answer)
 A: Who do you love best?
 B: It's Monday today, (no answer)

Just as a declarative sentence has truth conditions, we can postulate that a question-and-answer pair, considered as sentences, has **answer conditions**, that is, the conditions which must hold for the statement to constitute a true answer to the question.

The same trick can be played with imperatives, except that a linguistic

response is not always, perhaps not usually, what an imperative sets out to elicit. Generally what is required is an action (in the broadest sense, which covers such cases as *Don't move!*). In the case of a fully contextualized command, where a full **command meaning** is operative, a given action either counts as compliance or does not, that is, it has a **compliance value**, equivalent (in the relevant respects) to the truth value of a statement. What its compliance value is, is determined by the propositional content of the command. An imperative sentence has a set of compliance conditions, but no compliance value (how do you obey *Put it there*, without knowing what *it* is, or where *there* is?).

Let us now try to summarize and generalize. Declarative sentences, in general, do not, in themselves, make statements, interrogative sentences do not ask questions, and imperative sentences do not issue commands. These only result when the sentences are fully contextualized. Each of these, however, has propositional content, which governs what illocutions it can be used to perform when properly contextualized, that is, which statements can be made, questions asked, and commands issued, etc.

2.5 Logical properties of sentences

2.5.1 Logical relations between sentences

A number of logical relations between sentences make a not infrequent appearance in semantic discussions, especially in lexical semantics. They are related to, but not identical to, relations that logicians recognize between propositions. It is, however, necessary to bear in mind their logical basis, if confusion is to be avoided. Five relations will be recognized here: implication/entailment, equivalence, contrariety, contradiction, and independence.

2.5.1a Entailment

This and the following relations strictly speaking hold between propositions, and that is how we shall treat them to begin with; the connection with sentences will be dealt with later. Entailment is the relation which holds between the P and the corresponding Q items in the following:

<i>p</i>	<i>Q</i>
It's a dog.	It's an animal.
John killed the wasp. The wasp died.	
All dogs are purple. My dog is purple.	

Notice that we are assuming that these sentences express propositions; that is to say, the sentences are being used in a particular context with particular reference. We are further assuming that in the first pair *it* refers to the same entity in each case, and in the second pair, the same wasp and the same event are being referred to. To say that proposition **P entails** proposition **Q** means

that the truth of Q follows logically and inescapably from the truth of P, and the falsity of P follows likewise from the falsity of Q. So, in the first pair of sentences above, if it is true of some entity that it is a dog, then it follows ineluctably that it is an animal, and if it is not an animal, then there is no way it can be a dog. Similarly, in the second pair, if John killed some wasp, then we cannot avoid the conclusion that the wasp died, and if the wasp did not die, then it cannot be the case that John killed it.

Entailment, as used by linguistic semanticists, is to be distinguished from what logicians call **material implication**. A proposition P materially implies another proposition Q if and only if (henceforward *iff*) it is never the case that P is true and Q false. At first sight this seems to be essentially the same as entailment. However, there is a crucial difference: the definition of material implication makes no reference to the meanings of the propositions, merely to a relation between their truth values; entailment, on the other hand, (sometimes called **strict implication**) is essentially a relation between meanings. To illustrate this point, consider the propositions *Z is a dog* and *All bachelors are unmarried*. It can never be the case that the first is true while the second is false, so we have a case of material implication. But this is not entailment, because it is not a consequence of the meaning relations between the two propositions, but of the fact that the second proposition cannot under any circumstances (except by altering the meanings of the components) be false. On the other hand, although it is true that *It's a dog* materially implies *It's an animal*, because the former cannot be true while the latter is false, it is also the case that the former entails the latter, because the truth-value relation holds by virtue of the relation in meaning between *dog* and *animal*. Material implication is essentially of no interest to linguistic semantics, although the status of *All bachelors are unmarried* as being always true is of interest (see 2.5.2.1 below: *analytic expressions*).

Strictly speaking, entailment does not hold between sentences, because sentences do not have truth values. However, one frequently encounters in semantic texts statements to the effect that such-and-such a sentence entails some other sentence. This can be taken as a kind of shorthand for something slightly more complex. Saying that sentence S¹ entails sentence S² means that in any context where S¹ expresses a true proposition, S² also necessarily expresses a true proposition, provided that corresponding definite referring expressions in the two sentences are co-referential. Obviously, in the case of *It's a dog* and *It's an animal*, the two occurrences of *it* must refer to the same entity for the logical relation to hold, and in the case of *John killed the wasp* and *The wasp died*, we must be talking about the same wasp, and the time references must be the same.

Two other properties of entailment must be emphasized. The first is that the relation is not determined by context: it is context independent, since it depends entirely on the meanings of the constituents of the sentences. Consider a case where John has in front of him a box of coloured disks, in which

all the red disks are round in shape, and all the green disks are square. In such circumstances, the truth of *John picked a square disk from the box* follows inescapably from the truth of *John picked a green disk from the box*. But clearly this relation of truth values does not arise from relations between *green* and *square*, but from the context: it would in principle have been just as easy to have all the red disks square, and the green disks round. On the other hand, the relation between *It's a dog* and *It's an animal* is independent of any particular contexts.

The second property is that the truth of the entailed sentence must follow inescapably from the truth of the entailing sentence. It is not enough for it to be usually true, or even almost always true; it has to be unthinkable that it might be false. Consider the relation between *It's a dog* and (i) *It's a pet* and (ii) *It can bark*. Most dogs that most people encounter are pets, but there are such things as wild dogs, so the relationship is merely one of expectation. This is not entailment. Likewise in the case of (ii), most dogs can bark, but a dog with a defective larynx does not thereby cease to be a dog, so the relationship is not logically necessary. Only logically necessary, context-independent relationships count as entailment. (We shall modify this position in Chapter 3, but for the moment it stands.)

2.5.1.2 Equivalence

Propositional equivalence between two sentences can be straightforwardly defined as mutual entailment. That is, in effect, equivalent to saying that the two sentences always express the same proposition (provided, of course, that corresponding definite referring expressions are co-referential). The following are examples of equivalence:

John killed the wasp.	The wasp was killed by John.
The wasp is dead.	The wasp is not alive.
It began at 10 o'clock.	It commenced at 10 o'clock.

If it is true that John killed the wasp, then it is also true that the wasp was killed by John and if it is true that the wasp was killed by John, then it is also necessarily true that John killed the wasp; a parallel two-way entailment holds between the members of the other two pairs.

2.5.1.3 Contrariety

Contrary propositions may not be simultaneously true, although they may be simultaneously false. The following are examples:

John killed the wasp.	The wasp is alive.
John killed the wasp.	Mary killed the wasp.
This paint is red.	This paint is green.

It cannot be simultaneously true of some wasp both that John killed it and that it is still alive; on the other hand, if the wasp is actually dead, but it was

Bill who killed it, then both *John killed the wasp* and *The wasp is alive* are false. We can define this relation in terms of entailment, by saying that S^1 and S^2 are contraries iff S^1 entails not- S^2 , but not- S^2 does not entail S^1 (and vice versa). Thus, *TAŵ paint is red* entails *This paint is not green*, but *This paint is not green* does not entail *This paint is red*, since it might well be, for instance, yellow.

2.5.1.4 Contradiction

Contradictory propositions must have opposite truth values in every circumstance: that is, they cannot be either both true or both false. In any particular circumstance, one member of a contradictory pair must be true and the other false. The following sentences exemplify contradictory pairs:

The wasp is dead.	The wasp is alive.
John is still singing.	John is no longer singing.
No dogs are brown.	At least some dogs are brown.

If John is still singing, then it is false that he is no longer singing; to this extent, this is like contrariety. However, there is a crucial difference: if it is false that John is still singing, then it must be the case that he is no longer singing, and if it is false that he is no longer singing, then he must be still singing. We can define contradiction in terms of entailment, by saying that S^1 and S^2 are contradictories iff S^1 entails not- S^2 , and not- S^2 entails S^1 (and vice versa).

2.5.1.5 Independence

For some pairs of propositions, the truth values vary independently of one another: they may be both true, both false, or one true and the other false:

John is retired.	Mary is married.
It is Tuesday today.	Christmas day falls on a Wednesday this year.

The relations described in this section have an important role in the analysis of meaning relations between words, as we shall see in later chapters.

2.5.2 Analytic, paradoxical, and synthetic sentences

2.5.2.1 Analyticity

Analytic sentences are sentences which automatically express true propositions in any context, by virtue of the meanings of their constituent words and their arrangement. The following sentences are therefore analytical:

Bachelors are unmarried.
 John's uncle is a man.
 This proposition is either true or false.

2-S.2.2 Paradox

Paradoxical sentences automatically express false propositions:

Bachelors are married.
John's sister is a man.
This red paint is green.

2.5.2.3 Syntheticity

Synthetic sentences are those which express true propositions in some (conceivable) contexts (although they may be false of the world as we know it) and false ones in others (this is the normal kind of sentence used in communication):

John's sister is married.
This paint is green.
All dogs are brown.

(The last sentence is actually false, but it is not *logically* false; it is easy to imagine circumstances in which it would be true.)

2.6 Logical classes

2.6.1 Class relations

2.6.1.1 Identity

Two classes C^1 and C^2 are said to be **identical** if everything that belongs to C^1 also belongs to C^2 , and vice versa. Thus, the class of fathers and the class of male parents are identical, as are the class of pairs of spectacles and the class of pairs of glasses (on the relevant interpretation ofg/owes).

2.6.1.2 Inclusion

Class C^1 is said to include class C^2 if everything that is a member of C^2 is also a member of C^1 , but not vice versa. Thus, for instance, the class of animals includes the class of dogs, the class of aardvarks, etc. The set of dogs is described as a subclass of the set of animals, and the set of animals as a superclass of the set of dogs. (Inclusion is defined here so as to exclude identity; it can be defined so as to include identity.)

2.6.1.3 Disjunction

Classes C^1 and C^2 are said to be disjunct if no member of C^1 is also a member of C^2 . The class of cats and the class of aardvarks are disjunct in this sense, as are the class of red things and the class of green things.

2.6.1.4 Intersection

Classes C^1 and C^2 are said to intersect if they have some members in common, but each has members which do not belong to the other (i.e. complete inter-

section, or identity, is excluded here). The class of red things and the class of round things intersect in this fashion, as do the class of architects and the class of amateur musicians. The set of common members to two (or more) overlapping classes is often referred to as the intersection of the two (or more) classes.

2.6.1.5 Union

The combined set of members belonging to either of two (or more) classes (including overlapping classes) is called the union of the two (or more) classes. Thus the union of the class of dogs and the class of cats is constituted by the class of all entities which are either cats or dogs.

2.6.2 Class relations and propositional relations

There are obvious connections between the class relations described above and the propositional relations described earlier. For instance, the fact that *It's a dog* entails *It's an animal* is not unconnected to the fact that the class of dogs is a subclass of the class of animals (although the connection is not a necessary one unless we define the class of dogs as “the class of all possible dogs”). Some relations emerge more naturally within one approach than another. For instance, we gave no propositional parallel for class overlap. This could be called propositional independence, since the truth of either proposition in such a pair imposes no logical restraint on the truth value of the other, as in the case of *John is an architect* and *John is an amateur musician*. The relation of contradiction, on the other hand, emerges more naturally within the propositional approach. To find an equivalent of the relation between, say *dead* and *alive* within the class approach, we would have to say that the class of dead things and the class of alive things were (i) disjunct and (ii) exhaustive of a superclass of animate things.

2.6.3 Mapping

It sometimes happens that the members of one class have a relation of correspondence of some kind with one or more members of a parallel class. This type of correspondence is known as **mapping**. An example will make this clear. It is a well-known fact that a person's fingerprints are uniquely distinctive. If, therefore, we think of the class of persons and the class of fingerprints, there is a straightforward mapping relation between the two classes, in that each member of one class corresponds to a specific member of the other set. This is known as **one-to-one mapping**. Contrast this situation with the two classes `fATHERs` and `CHiLDren`. Every member of the `fATHERs` class corresponds to one or more members of the `CHiLDren` class, but every member of the `CHiLDren` class corresponds to a single specific member of the `fATHERs` class. Here we have **one-to-many** mapping between fathers and children, but **many-to-one** mapping between children and fathers. Yet another elementary mapping relation holds between the class of word forms and the class of meanings. If we

allow the possibility of synonymy, then some word forms (e.g. perhaps *begin* and *commence*) will map on to the same meaning, whereas other word forms (e.g. *bank*) will map on to more than one meaning. This is known as **many-to-many** mapping (see section 7.1.2 for a more detailed consideration of mapping between words and meanings).

2.7 Logical relations

Another useful set of concepts borrowed from logic are to do with relations between individual entities. The entities may be anything at all: objects, people, places, ideas. So can the relations be anything: “brother of”, “smaller than”, “has played string quartets with”, “logically depends on”. The logical properties of such relations can be grouped under four headings: transitivity, symmetry, reflexivity, and converseness.

2.7.1 Transitivity

A relation that is **transitive** is one such that if A is related in this specific way to B and B to C, then it follows inescapably that A stands in the relation to C. Suppose A, B, and C are people, and the relation is “is taller than”. Then if A is taller than B and B is taller than C, then A is necessarily taller than C. If a relation is **intransitive**, then if A stands in the relation to B and B to C, then it is logically impossible for A to stand in the relation to C. This is the case with “is the mother of”: if A is the mother of B and B the mother of C, then A cannot be the mother of C. A relation may be neither transitive nor intransitive; we shall call such a relation **non-transitive**. If John has played duets with Bill, and Bill has played duets with Tom, then we are not in a position to conclude anything regarding John’s musical relations with Tom.

2.7.2 Symmetry

A **symmetric** relation is one such that if A stands in a particular relation to B, then B necessarily stands in that same relation to A. For instance, if A is near to B, then B is near to A. If a relation is **asymmetric**, then if A stands in the relation to B, B cannot stand in the same relation to A. An example of an asymmetric relation is “is taller than”. Once again, it is useful to have a designation for relations that are neither symmetric nor asymmetric, such as “is sexually attracted by”; we shall label these **non-symmetric**.

2.7.3 Reflexivity

The property of reflexivity is not of great usefulness in semantic analysis: it is included for the sake of completeness. A relation is **reflexive** if something necessarily stands in that relation to itself. This includes most types of identity

relation such as “has the same name as”, “is the same age as”, etc. Again, we can recognize **irreflexive** relations like “is taller than”, and **non-reflexive** relations such as “knows the weight of”.

2.7.4 Converseness

Converseness is a relation between relations. Two relations are **converses** if one yields the same proposition as the other when the arguments are reversed. By this criterion, “above” and “below” are converses (perhaps more strictly “is above” and “is below”), because *A is below B*, assuming constancy of A and B, expresses the same proposition as *B is above A*. Other examples of (sentences expressing) converse relations are: *A is B's offspring! B is A's parent*, *A saw BIB was seen by A*, *A sold B to CIC bought B from A* (it is usual to disregard ‘automatic’ adjustments in grammatical realization, such as the change from *to* to *from* in the case of *buy* and *sell*). We have defined converseness in terms of two-way entailment between two sentences (e.g. *A is taller than B* and *B is shorter than A*). It is useful to have a relation defined on a one-way entailment. For instance, *A is B's doctor* entails *B is A's patient*, but the reverse entailment does not hold because other practitioners in the medical and para-medical field, such as dentists and speech therapists, also have patients. We shall say that *doctor* is a **semi-converse** of *patient*.

2.8 Quantification

2.8.1 Quantifiers

In standard first-order predicate calculus, propositional functions are constrained by **quantifiers**: these in effect limit the applicability of the predicate to the argument(s). Classical logic only has two quantifiers, the **existential quantifier** and the **universal quantifier**. The existential quantifier says something like this:

There exists at least one ‘x’ such that ‘x sneezed’.

This is typically expressed in logical notation as:

$\exists x$ (*sneezed* (x)).

This could be roughly translated as *Someone sneezed*. *A man sneezed* would go into this special logical language as:

There exists at least one individual x such that x is a man and x sneezed.

$\exists x$ (*sneezed* (x) & *man* (x)).

The universal quantifier corresponds roughly to the ordinary language *all*, *every*. Thus *Dogs are animals* would translate as:

For all x , x is a dog entails x is an animal.

$\forall x$ ($dog(x) \rightarrow animal(x)$).

2.8.2 Scope

In the sentence *Mary ruffled John's hair and kissed him again* we do not know without further contextual evidence, whether it was only Mary's kissing of John that was repeated, or the double action of ruffling the hair and kissing. This is an ambiguity of **scope**: we do not know how much of the previous sentence is included in the range of applicability of *again*. The term scope is usually used in connection with quantifiers: *again* is a kind of quantifier over events. An example involving a more traditional quantifier is: *Some women and foreigners must register with the police*. Here we are uncertain whether only some foreigners should register, or whether they all should (which would be the most natural interpretation of *Foreigners must register*). Reversing the order of constituents would remove the ambiguity: *Foreigners and some women must register*. The possibilities for variations of scope of this sort are tightly constrained by syntactic structure. (For a more detailed discussion of quantification, see Chapter 14, section 14.6.)

2.9 Use and mention

Consider the difference between (19) and (20):

(19) Snow has four letters.

Snow is a noun.

Snow is a natural kind term.

Snow is an English word.

Snow is easy to pronounce.

(20) Snow is white.

Snow damages crops.

Snow is frozen water.

The difference between these two sets is usually designated as a difference between **use** (here, of the word *snow*), as in (20), and **mention** (of the word *snow*), as in (19). In the sentences in (19) we are using the word form *snow* to identify a word of the language, and we then proceed to say something about that word; in the sentences in (20) we are using the word form to identify a substance in the world, prior to predicating something of it. A simple way of distinguishing the two is to apostrophize (or italicize) the language unit in question. If this makes a negligible effect on the meaning, then it is a case of mention:

Snow has four letters.

Snow is an English word.

**Snow* is white.

**Snow* damages crops.

Mention may involve any stretch of language:

Go to home is ungrammatical.

**Go to home* is ungrammatical.

This brings us to the end of our brief survey of useful logical notions, and provides an elementary toolkit which will be drawn upon, and sometimes further refined, as and when the occasion demands.

Discussion questions and exercises

1. Arguments and predicates

Mark the following predicates as one-, two-, three-, or four-place (1, 2,3,4) (think in terms of semantics rather than syntax):

*yawn steal thank pay be tall be taller than meet put imagine
day-dream cost understand explain*

2» Sentence, statement, utterance, and proposition

Of which of the above can the following be said?

X was inaudible.

X was uninformative.

X was false.

X was in a foreign accent.

X was ungrammatical.

X was insincere.

3. For each of the following pairs of sentences, say whether the propositional content of the members is the same or different:

- (i) (a) Take your hands off me! (said by a woman to a man)
(b) Take your filthy paws off me! (ditto)
- (ii) (a) I always get my bread from Gregg's, because it's cheaper.
(b) I always buy my bread from Gregg's, because it's cheaper.
- (iii) (a) Don't you find him rather skinny?
(b) Don't you find him rather thin?
- (iv) (a) Have you read the stuff he wrote about telepathy?
(b) Have you read the garbage he wrote about telepathy?

- (v) (a) She was there at the start of the race.
(b) She was there at the beginning of the race.
- (vi) (a) John hasn't turned up.
(b) John hasn't turned up yet
- (vii) (a) Old Joshua Hobblethwaite died last week.
(b) Old Joshua Hobblethwaite passed away last week.

4. In which of the following does the (a)-sentence entail the (b)-sentence? Are there any problems?

- (i) (a) X is a cat.
(b) X has four legs.
- (vii) (a) X is a pet.
(b) X is alive.
- 00 (a) X is a cat.
(b) X is an animal.
- (viii) (a) X is not dead.
(b) X is alive.
- (iii) (a) X is a cat.
(b) X is a quadruped.
- (ix) (a) X has stopped smoking.
(b) X doesn't smoke any more.
- (iv) (a) X is a quadruped.
(b) X has four legs.
- (x) (a) X taught YZ.
(b) Y learnt Z.
- (v) (a) X is a quadruped.
(b) X is an animal.
- (xi) (a) X killed Y.
(b) Y is not alive.
- (vi) (a) X is a pet.
(b) X is an animal.
- (xii) (a) X watched Y.
(b) Y was doing something.

5. Mark the propositional relationship between the members of the following pairs of sentences as either EQUIVALENCE, CONTRARIETY, **CONTRADICTION**, or **CONVERSENESS**:

- (i) (a) Proposition P is true.
(b) Proposition P is false.
- (ii) (a) John likes Mary.
(b) John dislikes Mary.
- (iii) (a) Mary agrees with the statement.
(b) Mary disagrees with the statement
- (iv) (a) Mary borrowed the book from John.
(b) John lent the book to Mary.
- (v) (a) John killed the wasp.
(b) The wasp is still alive.
- (vi) (a) John is not married.
(b) John is a bachelor.

6. Classify the following relations with regard to their TRANSITIVITY (i.e., as TRANSITIVE, INTRANSITIVE, or NON-TRANSITIVE) and their SYMMETRY (i.e., as SYMMETRIC, ASYMMETRIC, Or NON-SYMMETRIC):

*parent of ancestor of brother of related to sibling of friend of near to
to the right of far from resembles*

Suggestions for further reading

The treatment here has been very informal. A similar elementary treatment, but with more practical exercises, will be found in Hurford and Heasley (1983). Lyons (1995) develops the philosophical background more fully, but still at an elementary level. Those requiring initiation into logical formalization will find an accessible introduction in Allwood, Anderson, and Dahl, (1977). Lyons (1977) gives a more detailed treatment of many of the topics touched on here. Cann (1993), McCawley (1981), and Larson and Segal (1995) are only for those who are really serious about the application of logic to language.