Chapter 10

Formal Semantics

10.1 Introduction

In this chapter we look at the approach known as **formal semantics.** Although any approach might be formalized, this label is usually used for a family of denotational theories which use logic in semantic analysis. Other names which focus on particular aspects or versions of this general approach include **truth-conditional semantics, model-theoretic semantics,** and **Montague Grammar.**[1] As we shall see, another possible label might be **logical semantics.**

This approach elaborates further the use of truth, truth-conditions and logic discussed in chapter 4. There we reviewed the strategy of borrowing from logic the notion of **truth** and the formalism of **propositional logic** to characterize semantic relations like entailment. In this chapter we shall see how further tools from logic can be used to help characterize aspects of sentence-internal semantics. In discussing formal semantics we touch on an important philosophical divide in semantics: between **representational** and **denotational** approaches to meaning. In chapter 9 we saw examples of the representational approach: for semanticists like Jackendoff semantic analysis involves discovering the conceptual structure which underlies language. For such linguists the search for meaning is the search for mental representations. Formal semanticists on the other hand come at meaning from another angle: for them a primary function of language is that it allows us to talk about the world around us. When communicating with others and in our own internal reasoning we use language to describe, or model, facts and situations. From this perspective, understanding the meaning of an utterance is being able to match it with the situation it describes. Hence the search for meaning, from the denotational perspective, is the search for how the symbols of language relate to reality.

How is this relation characterized? Formal semanticists employ the **correspondence theory** of truth discussed in chapter 4. Speakers are held to be aware of what situation an utterance describes and to be able to tell whether the utterance and the situation match up or **correspond.** Thus knowing the meaning of an English sentence like *It's raining in Belfast* involves understanding what situation in the world this sentence would correspond to, or fit. A successful match is called **true;** an unsuccessful match is **false.** Another way of describing this is to say that the listener who understands the sentence is able to determine the **truth conditions** of the uttered sentence, that is, know what conditions in the world would make the sentence true. In the basic version of this approach used in logic there are no *almosts* or *nearly*^'. an utterance either describes a situation, and is therefore true of that situation, or not, in which case it is false. See, for example, the characterization from a logic text, Bradley and Swartz (1979: 11):

10.1 The account of truth which we are espousing here has been described variously as 'the Correspondence Theory', 'the Realist Theory', or eVen 'the Simple Theory' of truth. In effect, it says that a proposition, P, is true if and only if the (possible) states of affairs . . . are as P asserts them to be. It defines 'truth' as a property which propositions have just when they 'correspond' to the (possible) states of affairs whose existence they assert. It is a 'realist' theory insofar as it makes truth a real or objective property of propositions, i.e. not something subjective but a function of what states of affairs exist in this or that possible world. And it is a 'simple' theory of truth insofar as it accords with the simple intuitions which most of us - before we try to get too sophisticated about such matters - have about the conditions for saying that something is true or false.[2]

Some objections to this might quickly occur to a linguist seeking to borrow these notions to describe natural languages. On a practical descriptive level, this characterization seems to apply just to statements, since intuitively it is hard to see how other utterance types like questions and orders can be viewed as descriptions of situations. Yet, as we saw in our discussion of speech acts in chapter 8, many utterances are not statements. On a more general level the idea of correct or incorrect matches seems to remove the subjectivity of the speaker. We saw in chapter 5 that the certainty shown by

a statement might be just one of a range of speaker attitudes to, or degrees of confidence in, a proposition. We described such ranges with terms like **modality** and **evidentiality.** In section 10.8 we discuss how formal approaches might take account of such notions.

Formal semanticists have to meet these and related objections to the extension of logical mechanisms to ordinary language. Nonetheless, this approach has become one of the most important and liveliest in the semantics literature. Why is this? We can perhaps outline at this preliminary stage a number of advantages. One great advantage comes from using logical expressions as a semantic metalanguage. It enables semanticists to import into linguistics the economy and formality of the traditional discipline of logic and the benefits of the long struggle to establish mathematics and logic on common principles.[3] Logicians try to make as explicit as possible both the relations between logical symbols and what they represent and the effects of combining symbols. Consequently logic, as a potential semantic metalanguage, has the important advantage of precision.

Denotational approaches, if successful, have another advantage: they escape the problem of circularity discussed in chapter 1. We raised the problem that if we interpret English in terms of a metalanguage, another set of symbols, then we have just translated from one language to another. This second language then needs a semantics, and so on. As we shall see, formal semanticists do translate a natural language like English into a second, logical language, but this translation is only part of the semantic analysis. This logical language is then semantically grounded by tying it to real-world situations. The aim of a denotational approach is not just to convert between representations: it seeks to connect language to the world.

There are other less obvious advantages claimed for such theories: it has been suggested, for example by Chierchia and McConnell-Ginet (1990), that denotational approaches allow us to see more clearly the connection between human languages and the simpler sign systems of other primates like vervet monkeys, baboons and chimpanzees. These systems are clearly referential: primates often have distinct conventional signs for different types of predators like eagles, snakes or big cats.[4] Perhaps this basic matching between a symbol and entities in the environment was the starting point for human languages.

Whatever the advantages to this approach, we should mention one temporary, practical disadvantage for students new to the theory: this is a very technical and highly formalized approach. Employing the tools of logic means having to become familiar with them and this involves a substantial expenditure of time and effort. Beginners will not see a return on this investment, in terms of improved semantic analyses of real language, very quickly. For an introductory survey like this one, this poses problems of coverage. How much of this large and complicated technical apparatus can we cover in a chapter like this? Our proposal is to sketch in the basic features of the approach without too steep an immersion into mathematical formulae. In particular we will not investigate the formal proofs that a logical language must support. We mention some book-length introductions to this approach in our suggestions for further readings at the end of the chapter which will allow the interested reader to pursue these topics more fully.

10.2 Model-Theoretical Semantics

Much of the investigation of logic and natural language semantics has been conducted by philosophers, logicians and mathematicians: for example the predicate logic we describe in this chapter derives largely from the work of the logician and mathematician Gottlob Frege,[5] the notion of truth owes much to Alfred Tarski (1944, 1956), and much of the recent and contemporary debate has been undertaken by philosophers like Donald Davidson (e.g. 1980, 1984). For many linguists, interest in this approach was sparked by the work in the 1960s of the logician Richard Montague, mentioned earlier. As we shall see, an important element in this theory is a **model,** a formal structure representing linguistically relevant aspects of a situation. Consequently one term for Montague's work and similar approaches is **model-theoretical semantics.** The application of this approach to linguistic description by linguists and computer scientists has led both to further development of the model-theoretical approach and the emergence of a number of related but distinct approaches, like **situation semantics** (Barwise and Perry 1983) and **discourse representation theory** (Kamp and Reyle 1993). Since, our discussion will remain at an introductory level, we begin by outlining a kind of embryonic model-theoretic approach. Our description will be influenced by Montague Grammar but we will not attempt an introduction to this theory here[6] (see Montague 1974, Dowty, Wall and Peters 1981). In such an approach semantic analysis consists of three stages: firstly, a translation from a natural language like English into a logical language whose syntax and semantics are explicitly defined. Secondly, the establishment of a mathematical model of the situations that the language describes. Thirdly, a set of procedures for checking the mapping between the expressions in the logical language and the modelled situations. Essentially these algorithms check whether the expressions are true or false of the modelled situations. Each of these three stages can throw light on the semantic capabilities of natural languages.

We look at these stages in order: we discuss the translation in section 10.3, where we use English as our example and we concentrate on the syntax of the logical metalanguage. We discuss models and mapping algorithms in 10.4-5, where the emphasis is on adding a semantics to the metalanguage. In 10.6 we discuss word meaning in formal semantics.

Subsequently we review some key areas where this basic model has been extended to reflect more accurately the semantics of natural languages. In 10.7 we look at quantifiers in more detail; in 10.8 we discuss intensionality;

and in 10.9 we look at an approach which takes account of the dynamic nature of communication, discourse representation theory (DRT).

10.3 Translating English into a Logical Metalanguage

10.3.1 Introduction

As we have said, the first stage of this semantic analysis consists of translation. The basic idea is that we can translate from a sentence in an individual language like English into an expression in a universal metalanguage. One such metalanguage is **predicate logic.** As mentioned in chapter 4, predicate logic builds on the investigation of sentence connectives in propositional logic and goes on to investigate the internal structure of sentences, for example the truth-conditional effect of certain words like the English quantifiers aZZ, *some, one,* etc. In chapter 4 we briefly introduced a set of logical connectives which parallel in interesting ways some uses of English expressions like *and, or, if. . . then,* and *not.* These connectives are summarized in 10.2 below; for each connective the table gives its symbol, an example of its syntax, i.e. how it combines with sentence constants **p, q** etc., and an approximate English equivalent:

Connectives in propositional logic

Connective	**Syntax**	**English**
—i	**-p**	it is not the case that **p**
A	**P** A **q**	p and q
V	**p** v **q**	p and/or q
v_e	**P** v_e **q**	**p** or **q** but not both
—»	**P -> Q**	if **p,** then **q**
=	**p = q**	p if and only if q

We will be using these connectives in our translations into predicate logic, which we begin in section 10.3.2.

10.3.2 Simple statements in predicate logic

If we begin with simple statements like 10.3 and 10.4 below:

10.3 Mulligan is sleeping.

10.4 Bill smokes.

we can identify a subject-predicate structure where the subject is a referring expression *(Mulligan, Bill)* and the predicate tells us something about the subject *(is asleep, smokes')*. The predicate logic assigns different roles to these two elements: the predicate is treated as a skeletal function which requires the subject argument to be complete. Our first step is to represent the predicate by a capital **predicate letter,** e.g.:

10.5 is asleep: *A*
smokes: 5

The subject argument can be represented by a lower case letter (usually chosen from *a* to *t* and called an **individual constant),** e.g.:

10.6 Mulligan: *m*
Bill: *b*

The convention is that predicate logic forms begin with the predicate, followed by the subject constant. Thus our original sentences can be assigned the representations in 10.7:

10.7 Mulligan is asleep: *A(m)*
Bill smokes: *S(b)*

If we want to leave the identity of the subject unspecified we can use **variables** (lower case letters from the end of the alphabet: w, x, *y, z),* e.g.:

10.8 *x* is asleep: *A(x)*
y smokes: \SCy)

As we shall see later, these variables have a special use in the analysis of quantifiers.

We have been looking at the representation of intransitive sentences. The verbs in transitive sentences like 10.9 below require more than one nominal:

10.9 Bill resembles Eddie.
Tommaso adores Libby.

These predicates are identified as relations between the arguments and represented as follows:

10.10 Bill resembles Eddie: *R(b, e)*
Tommaso adores Libby: *A(t, I)*

Other relational sentences will be represented in the same way, e.g.:

10.11 Pete is crazier than Ryan: *C(p,* r)

Note that the order of constant terms after the predicate letter is significant: it mirrors English sentence structure in that the subject comes before the

object. Three-place relations are of course possible; we show an example with its logical translation below:

10.12 Fatima prefers Bill to Henry: *P(f, b, h)*

In our examples so far we have included the English sentence and the logical translation. Alternatively, we can keep track of what the letters in the logical form correspond to by providing a key, e.g.:

10.13 *P(f, b, h)*
Key: *Pt* prefer
ft Fatima
bt Bill
ht Henry

Our notation so far can reflect negative and compound sentences by making use of the connectives shown earlier in 10.2, for example:

10.14 Maire doesn't jog: *—\}(m)*

10.15 Fred smokes and Kate drinks: *S(f)* A *D(k)*

10.16 If Bill drinks, Jenny gets angry: *D(b) —> A(j)*

We might also wish to translate sentences containing **relative clauses** like *(the student) zvho passed the exam, (the dress) that she wore,* etc. We can represent complex sentences containing relative clauses by viewing them as a form of conjunction, i.e. by using A 'and', as in 10.17-19 below:

10.17 Carrick, who is a millionaire, is a socialist: *M(c)* A 5(C)

10.18 Emile is a cat that doesn't purr: *C(e) A —iP(e)*

10.19 Jean admires Robert, who is a gangster: *A(j, r)* A *G(r)*

In the next section we extend the logic further to cope with quantified noun phrases.

10.3.3 Quantifiers in predicate logic

One important feature of natural languages that formal semanticists have to deal with in their translation into logical form is **quantification.** All languages have strategies for allowing a proposition to be generalized over ranges or sets of individuals. In English for example quantifiers include words like *one, some, a few, many, a lot, most* and *all.* We can look at a simple example. Let's say that we want to predicate the verb phrase *wrote a paper* of various members of a class of students. We could assert this predicate will be true of (at least) one member., by saying 10.20 below:

10.20 A student/Some student wrote a paper.

or vary the range of its applicability, as below:

10.21 a. A few students wrote a paper.
b. Many students wrote a paper.
c. Most students wrote a paper.
d. All students wrote a paper.
e. Every student wrote a paper.

We could also deny it applies to any of them by using:

10.22 No student wrote a paper.

The simple logical representation we have developed so far isn't able to reflect this ability to generalize statements over a set of individuals. One way to do' this is to follow a proposal of Frege's that statements containing quantifiers be divided into two sections: the quantifying expression which gives the range of the generalization; and the rest of the sentence (the generalization), which-qyill have a place-holder element, called a **variable,** for the quantified nominal. We can show how this approach works for the quantifiers *ally everyy somey* and *no₃* though as we shall see in section 10.7 later the other quantifiers in example 10.21 will require a different account. To show this we look first at the quantifiers *all* and *every.* Both of these English quantifiers are represented in predicate logic by the **universal quantifier,** symbolized as V. We can as an example 10.21e above. This will be given the representation 10.23a below, which can be read as 10.23b:

10.23 a. Vx (S(x)lT(x,/>))
b. For every thing x, if x is a student then x wrote a paper.

The universal quantifier establishes the range by fixing the value of x as every thing; the expression in parentheses is the generalization. By itself the generalization is an incomplete proposition, called an **open proposition:** until the value of x is set for some individual(s) the expression cannot be true or false. As we shall see, the quantifier serves to set the value of x and close the proposition. Expressions with the universal quantifier can be paraphrased in English by *all* or *every* as in *All students wrote a paper* or *Every student wrote a paper* in 10.23.[7]

We can see that the quantifier phrase can be associated with different positions in the predicate if we compare 10.24a and b below:

10.24 a. Every student knows the professor: Vx (S(x) —» K(x, p))
b. The professor knows every student: Vx (S(x) *K(py* x))

Here the logical representations emphasize more than English does that both 10.24a and b are predicating something about all of the students. The relationship between the quantifier phrase and the rest of the formula is described in two ways: the quantifying expression is said to **bind** the variable in the predicate expression; and the predicate expression is said to be the **scope** of the quantifier.

Next we turn to the quantifier *some* in example 10.20. *Some* is represented in predicate logic by the **existential quantifier,** symbolized as 3. We can thus translate our example as 10.25 below:

10.25 3x (S(x) A P(s, e))
There is (at least) one thing x such that x is a student and x wrote a paper.

We can paraphrase such expressions in English by using noun phrases like *a studenty some studenty* and *at least one student.* The existential quantifier can also be associated with different positions in the predicate:

10.26 (At least) One student kissed Kylie: 3x (S(x) A K(X, &))

10.27 Kylie kissed (at least) one student: Sx (S(x) A *K(ky* x))

Once again the existential quantifier is said to bind the variable and the predicative expression is described as the scope of the quantifier.

The English determiner *no* can be represented by a combination of the existential quantifier and negation, as shown below:

10.28 -3x (S(x) A lT(x,p))
It is not the case that there is a thing x such that x is a student and x wrote a paper. There is no x such that x is a student and x wrote a paper.

This corresponds to the sentence *No student wrote a paper.* Another way of representing this is by using the material implication:

10.29 Vx (S(x) -> -nP(x, *e})*
For every thing x, if x is a student then it is not the case that x wrote a paper

With the introduction of these quantifiers we can now summarize the syntax of the predicate logic so far. The syntax includes the vocabulary of symbols in 10.30 below and the rules for the formation of logical formulae in 10.31:

10.30 The symbols of predicate logic

Predicate letters:	*A, B,* C, etc.
Individual constants:	*a, b, c,* etc.
Individual variables:	*x, y, z,* etc.
Truth-functional connectives:	A, V, v_e, =
Quantifiers:	V, 3

10.31 The rules for creating logical formulae

a. Individual constants and variable are terms.
b. If A is an n-place predicate and tj . . . are *n* terms, then A(tj ... t_n) is a formula.[8]
c. If § is a formula, then —1<|) is a formula.
d. If (j) and y are formulae, then (([) A \|/), ((|) V y), (<|) v_e y), ((|) —» y), (()) = y) are all formulae.
e. If $ is a formula and x is a variable, then Vx(|), and 3x(j) are formulae.

We can add to these rules the convention that the outer parentheses of a complete formula can be omitted, i.e. instead of writing (<$> A y), we can write <|) A \p.

10.3.4 Some advantages of predicate logic translation

The predicate logic we have been looking at is used by logicians to demonstrate the validity of arguments and reasoning. Thus in addition to a syntax and semantics, the logical languages requires rules of inference. This however is a topic we will not pursue here. From a linguist's perspective there are a number of advantages to the representations we have introduced. We can take as an example the way that the representation of quantifiers, as introduced above, clarifies some ambiguities found in natural languages. One of these is **scope ambiguity,** which can occur when there is more than one quantifier in a sentence. For example the English sentence 10.32a below has the two interpretations paraphrased in 10.32b and c:

10.32 a. Everyone loves someone.
b. Everyone has someone that they love.
c. There is some person who is loved by everyone.

Version 10.32b involves a many-to-many relationship of loving, while version 10.32c involves a many-to-one relationship. While the English sentence is structurally ambiguous between these two interpretations, the difference is explicitly shown in predicate logic by the ordering of the quantifiers. The interpretation in 10.32b is represented by the formula in 10.33a and that in 10.32c by 10.33b below:

10.33 a. Vx3y (L(x, y))
b. 3jA7x (L(x, y))

The formula in 10.33a says that for every person x, there is some person *y* that they love. The universal quantifier comes leftmost and therefore contains the existential quantifier in its scope. This situation is described as the universal quantifier having **wide scope. In** 10.33b we have the reverse: the existential quantifier contains the universal in its scope and therefore takes wide scope. Thus the scope of one quantifier may be contained within the scope of another.

Negative words, like English *not,* also display scope over a predication and a second advantage of this type of representation is that it allows us to disambiguate some sentences which contain combinations of quantifiers and negation. The sentence *Everybody didn't visit Limerick,* for example, can have the two interpretations given in 10.34 and 10.35 below, where we give a paraphrase in b and the predicate logic translation in c:

10.34 a. Everybody didn't visit Limerick.
b. For every person x, it's not the case that x visited Limerick.
c. Vx -.(F(x, Z))

10.35 a. Everybody didn't visit Limerick.
b. It's not the case that every person x visited Limerick.
c. —.Vx (F(x, Z))

As we can see, the ambiguity is clearly distinguished in the predicate logic translations. In 10.34c the universal quantifier has wide scope over the negative connector while in 10.35c the negative has wide scope over the universal quantifier.

These examples have shown some of the advantages of semantic clarity gained by the translation into predicate logic. In fact though, as we mentioned earlier, the real purpose of this translation is to allow a denotational semantic analysis to be carried out. In the next section we look at how this logical representation is given a semantics.

10.4 The Semantics of the Logical Metalanguage

10.4.1 Introduction

As we have said, the aim of this approach is to devise a denotational semantics. Clearly our first stage alone is not such a semantic analysis. Translating from an English sentence into a logical formula is not enough: we then have to relate this second set of symbols to something outside - the situation described. To do this we need to add three further elements:

10.36 1 a **semantic interpretation** for the symbols of the predicate logic;
2 a **domain:** this is a model of a situation which identifies the linguistically relevant entities, properties and relations; and
3 a **denotation assignment function:** this is a procedure, or set of procedures, which match the logical symbols for nouns, verbs, etc. with the items in the model that they denote. This function is also sometimes called a **naming function.**

The domain and naming function are together called a **model.** We look at each of these constituents in turn.

10.4.2 The semantic interpretation of predicate logic symbols

We can adopt a simple denotational theory of reference, as discussed in chapter 2, for the units of the predicate logic. We will identify three such units for discussion: whole sentences, constant terms, and predicates and we will use some simple **set theory** notions to help us define denotation.[9]

Sentences

Following the correspondence theory of truth" we will take the denotatum of a whole sentence to be the match or lack of match with the situation it describes. A match will be called **true (T),** also symbolized by the numeral **1.** A mismatch will be called **false (F),** symbolized by the numeral **0.** So using a variable sAfor situations, we might say 'a sentence p is true in situation and symbolize it as **[p]°** = 1. Here we use square brackets to symbolize the denotatum of an expression, so $[x]^w$ means the denotatum of *x* in the situation *v.* Thus the notation $\mathbf{[p]^u = 1}$ means 'the denotatum of **p** in *v* is **true'.** By contrast the expression $\mathbf{[p]^v = 0}$ will be read as 'the denotatum of **p** in *v* is **false'** or, equivalently, 'the sentence **p** is false in situation *v?* Since, as we have acknowledged, meaning is compositional, we want the truth-value of a sentence to be determined by the semantic value of its parts: the nouns, verbs, connectives, etc. of which it is constructed.

Individual constant terms

We will assume the denotation of individual constant terms to be individuals or sets of individuals in the situation. So if we adopt as our situation the 1974 world heavyweight title fight between Muhammad Ali and George Foreman in Zaire, we could use an individual constant term *a* to denote Ali, another individual constant *f* to denote Foreman and a third, r, to denote the referee in this situation *v.*

Predicate constants

We will assume that predicate constants, abbreviated with capital letters, P, Q, *R* etc., identify sets of individuals for which the predicate holds. Thus a one-place predicate like *be standing* will pick out the set of individuals who are standing in the situation described. This can be described in a set theory notation as either *{x* | . . . } or {x: . . .}, both of which can be read as 'the set of all x such that. . .'. So a notation like {x: x is standing in *v}* can be read as 'the set of individuals who are standing in situation *v*\

Two-place predicates identify a set of ordered pairs: two individuals in a given order. Thus the predicate *punch* will pick out an ordered pair where the first punches the second in *v* represented in set theory terms as: {<x, *y>: x* punches *y* in *v}*. Similarly a three-place predicate like *hand to* will identify a 3-tuple {<x, y, *z>'. x* hands *y* to *z* in *v}.*

10.4.3 The domain

The domain is a representation of the individuals and relationships in a situation, which we will continue to call *v.* Let's invent an example by imagining a situation in the Cavern Club, Liverpool in 1962 where the Beatles are rehearsing for that evening's performance. If we use this as our domain, let's say we can identify several individuals in the situation: the Beatles themselves, John, Paul, George and Ringo, their manager Brian Epstein and one stray fan we'll call Bob. In the format we are using here we will say that the situation *v* contains a set of individuals, U, such that in this case U = {John, Paul, George, Ringo, Brian Epstein, Bob}.

10.4.4 The denotation assignment function

This function matches symbols from the logical representation with elements of the domain, according to the semantic nature of the symbols. For our simple example, we can divide its work into two parts: (a) the matching of individual constant terms with individuals in the situation *v,* and (b) the matching of predicate constants with sets of individuals in *v.*

Matching individual constant terms

The assignment is- a function, which we can symbolize as F(x). This function will for any symbol x of the logical formula always returns its **extension** in the situation. Thus we can establish a matching for individual constant terms as follows:

10.37 Assignment of individual constant terms
F(j) = John
F(p) = Paul
F(^) = George
F(r) = Ringo
F(e) = Brian Epstein
F(6) = Bob

In other words, the individual constant j denotes the entity John in the situation f, *p* denotes Paul, and so on.

Matching predicate constants

Our function F(x) will return the extensions of predicates, as described a little earlier on for the semantics of predicates. Thus the function will return individuals, ordered pairs or 3-tuples, depending on the type of predicate. For our current example the matching will be as follows:

10.38 Assignment of predicate letters
F(B) - was a Beatle = {John, Paul, George, Ringo}
F(M) = was a manager = {Brian Epstein}
F(F) = was a fan = {Bob}
F(S) = sang = {John, Paul}
F((7) = played guitar = {John, Paul, George}
F(D) = played the drums = {Ringo}
F(J) = joked with = {<John, George>}
F(7) = idolized = {<Bob, John>, <Bob, Paul>, <Bob, George>, <Bob, Ringo>}

Thus the extension of *J 'joked with"* in the situation is the set of the ordered pair, John and George.

At this point then we have defined the semantic (denotational) behaviour of some of the logical constituents and established a model, which we take to be a combination of a domain and the assignment function. Such a model is often schematically described as $M_n = <U_n, F_n>_3$ where *M* - the model, *U* - the set of individuals in the situation, and F is our denotation assignment function. The subscript n (for 1, 2, 3 ... n) on each element relativizes the model to one particular situation. So we can identify our situation as $M_j = <Z7_{13}$ Fj>. For a different situation we would need a second model, $Af_2 = <U_{23} F_2>$, and so on.

Next we need to have some evaluation procedure to reflect a listener's ability to evaluate a sentence's truth-value relative to a situation. Basically this means a set of algorithms to check whether a given sentence is true or not of the situation. We outline a simple informal version of this in the next section.

10.5 Checking the Truth-Value of Sentences

As we mentioned earlier, our procedures for checking the truth-value of a sentence must reflect the compositionality of meaning. If this is done correctly, then we will have shown something of how the constituents of a sentence contribute to the truth-value of the whole sentence. To keep our discussion within bounds, we will look at this procedure for just three basic types of sentence: a simple statement, a compound sentence with A 'and', and sentences with the universal and existential quantifiers, V and 3.

10.5.1 Evaluating a simple statement

If we take our model M_{ls} we might construct some relevant sentences in predicate logic as in 10.39 below, some of which are true of Afj and some of which are false:

10.39
a. D(r)
b. 6(0
c. 0
d. G(p)
e. SO)

The reader may routinely translate these back into English, for example, 10.39a as *Ringo played the drums.** etc. Let's take 10.39e as an example and test its truth-value in Afj. The procedure for checking if *S(j)* is true is based on the denotational definitions we gave earlier and can be schematized as in 10.40 below:

10.40 $[S(;)]^{MI} = 1$ iff $[;]^{M‘}$ e $[5]^{MI}$

This rather forbidding schema employs various elements of our notation so far and can be paraphrased in English as in 10.41:

10.41 The sentence *John sang* is true if and only if the extension *otjohn* is part of the set defined by *sang* in the model

Now to check this we have to check the extensions returned by the denotation assignment function for the individual constant *j* and the predicate constant 5 to see if F/j) G F/S). We know from our model and assignment that:

10.42 FJ(J) = John

and we also know:

10.43 Fj(5) = {John, Paul}

So since it is clearly true that John G {John, Paul}, then our sentence is true, i.e. schematically $[*S(j)]^{MI} = 1$.

10.5.2 Evaluating a compound sentence with A 'and'

Evaluating a compound sentence follows basically the same procedure we have just outlined. Let's take as an example sentences containing A 'and'. We can create such sentences as 10.44 below for our model

10.44 a. *S(j) A S(p)*
b. *J(j. g)* * J(r, *b)*
c. *M(e) A F(b)*
d. S(j) A *I(b, e)*

Once again 10.44 contains some true and some false sentences for Af_P We can take as our example 10.44d, S(j) A *I(b, e).* To evaluate any compound sentence **p** A **q** we first establish the independent truth-value of **p** and then of q. Then we evaluate the effect of joining them with A. The truth-functional effect of A was given in the form of a truth table in chapter 4: essentially a compound with A is only true when **p** is true and **q** is true. In the format we are using here this behaviour can be expressed as in 10.45 below:

10.45 Truth behaviour of A
[p A q] = 1 iff [p] = 1 and [q] = 1

In effect 10.45 says that both conjuncts must be true for the compound to be true. If we turn again to our example 10.44d above, we can run through the procedure for evaluating its truth-value relative to M_x. For this particular sentence and model, the behaviour of A can be expressed as below:

10.46 $[£(;)$ A $I(b>£)]^{Ml}$ = 1 iff $[S(j)]^{Af‘}$ = 1 and $[I(b.\ e)]^{M>}$ = 1

That is, both of these conjuncts have to be true in M_v for the sentence to be true in Afj. Well, we already know from our discussion of simple statements that S(j) is true, so we can go on to evaluate 7(6, *e)* in the same way. The relevant rule is 10.47:

10.47 $[Z(Z>_3\ e)]^{M‘}$= 1 iff [<«>, e $[7]^{M‘}$

We can paraphrase this in English as 10.48 below:

10.48 The sentence *Bob idolized Brian Epstein* is true if and only if the extension of *Bob* and the extension of *Brian Epstein* are an ordered pair which is part of the set defined by *idolized* in the model M_x.

Thus *I{b.* e) = 1 iff <Bob, Brian Epstein> e F_1 (2). We can easily check this. The denotation assignment function will give the relevant values for this sentence as in 10.49 below:

10.49 a. F/6) = {Bob}
b. Fj(e) = {Brian Epstein}
c. *Fi(T)* = {<Bob, John>, <Bob, Paul>, <Bob, George>, <Bob, Ringo>}

We can see that the ordered pair <Bob, Brian Epstein> is not part of the set defined by the predicate *I,* i.e. <Bob, Brian Epstein> £ {<Bob, John>, <Bob, Paul>, <Bob, George>, <Bob, Ringo>}, so our sentence 7(6, *e)* is **false.**

Since our first conjunct S(j) is true and our second, Z(7>, *e).* false, then by the rule in 10.46 the whole sentence *S(j)* A 7(6, *e)* is false. This evaluation procedure may see rather laborious as we step through it in this simple way, but the importance for semantic analysis is that the procedure is explicit, is based on our semantic definitions of logical elements and the well-proven behaviour of logical connectives, and is productive: it can be applied in the same way to more and more complicated structures. The other truth-functional connectives can be treated in the same way as A by reflecting their respective truth-functional behaviours, described in truth tables in chapter 4, in rules paralleling 10.45 above, thus allowing the evaluation of sentences containing -i ‘not’, v ‘or’, -» ‘if. . . then’, etc.

10.5.3 Evaluating sentences with the quantifiers V and 3

The same procedure can, with some modification, be used to evaluate sentences with the universal and existential quantifiers, V and 3. We won’t give the step-by-step detail here but we can outline the spirit of the approach, using a different example. Let’s imagine a sad situation of a house that has three cats *(Tom. Felix* and *Korky)* and just one mouse *(Jerry).* Tom and Felix hunt Jerry but Korky does not. Without setting up a model for this we can see that one might say of the situation the (false) statement below:

10.50 Everyone hunts Jerry. Vx *(H(x. j))*

As we saw earlier the quantifier phrase Vx expresses the range of the generalization 77(x, *j)* and the quantifier binds the variable x. The evaluation procedure can exploit this structure as follows. We reflect the meaning of V, *every,* by establishing the rule that a sentence with this quantifier is true if the generalization is true for **each** denotation of *x.* otherwise false. Thus we need to test the truth of the expression x *hunts Jerry* for each individual in the situation that x can denote.

We already have a function F_n that matches individual constant terms with their denotation in the situation; we need another function, let’s call it g„, to do the same for variables. Such a function would successively match each individual in the situation with the variable x. In this situation the following are possible matchings:[10]

10.51 a. x - Tom
b. x = Felix
c. x = Korky

All we need to do then is test the generalization with each value for x, i.e. use the procedure we used for simple statements earlier to evaluate each of the following versions:

10.52 a. *x* = Tom: is *H*(x_3 j) true/false?
b. *x* = Felix: is *H*(x_s *j)* true/false?
c. *x* = Korky: is *H*(X_3*j)* true/false?

Once again, we won't step through the evaluation for each. Since of course 10.52a and b are true and 10.52c is false of this situation, then we know that the universal quantifier sentence Vx (H(x,j)) is false.

Sentences containing the existential quantifier 3 can be evaluated in the same way, except that the rule for this quantifier is that if the generalization is true of **at least one** individual in the range, the quantified sentence is true. Let's take for example, the sentence 10.53 below:

10.53 Some cat hunts Jerry. 3x (C(x) A *H*(X_3 J))

Once again the possible denotations for x are the three cats and we would evaluate the truth of C(x) A *H*(X_3*J)* with x set for the three values in 10.51:

10.54 a. x = Tom: is C(x) A *H*(X_3*j)* true/false?
b. x =$_v$Felix: is C(x) A *H*(X_3*j)* true/false?
c. x = Rorky: is C(x) A *H*(X_3*f)* true/false?

The truth table for A will tell us that 10.54a and 10.54b are true in our situation, while 10.54c is false. Consequently the existential quantifier rule that at least one must be true is satisfied and the sentence 3x (C(x) A *H*(X_3 7*)) is true.

We have of course only sketched this evaluation procedure for quantifiers; for example we haven't given the formal detail of the function g,, which assigns denotations to variables. For a fuller account of this approach see Chierchia and McConnell-Ginet (2000: 126ff.).

We have outlined in this section a denotational semantics for the predicate logic translations we introduced in 10.3. As we have observed, such a semantics has a number of advantages. From a methodological point of view, it has the advantages of being formal and explicit.[11] More generally it adopts the denotational program of relating utterances to specific situations. The semantics also embodies certain key features of natural languages in that it is compositional and productive; and more specifically, it allows the identification of individuals, sets of individuals and relations and, in a so far limited way, allows quantification. In the next section we look at how this approach accounts for word meaning.

10.6 Word Meaning: Meaning Postulates

As we have seen, when it comes to dealing with word meaning, the model-theoretic approach we have been looking at places great emphasis on the denotational properties of words. This is consistent with this approach's general assumption that the focus of semantic enquiry is sentence meaning: the idea is that the meaning of words is something best not pursued in isolation but in terms of their contribution to sentence meaning. Thus most formal approaches define a word's meaning as the contribution it makes to the truth-value of a sentence containing it.

However, the original structuralist position that words gain their significance from a combination of their denotation (reference) and their sense still seems to have force. We can return to our example from chapter 3: that if an English speaker hears 10.55 below, he knows 10.56:

10.55 I saw my mother just now.

10.56 The speaker saw a woman.

As we saw in chapter 3, speakers and hearers have knowledge about many kinds of sense relations between words, or what we termed **lexical relations.** The question for formal approaches is how to capture this lexical knowledge in a format compatible with the model-theoretical approach we have been looking at. One solution is to use **meaning postulates,** a term from logic (see Carnap 1952), and an approach advocated by J. D. Fodor et al. (1975) and Kintsch (1974).

The meaning postulates approach would recognize that 10.56 follows automatically from knowledge of 10.55 but rather than state this in terms of components of meaning of either word, this approach simply identifies this relationship as a form of knowledge,[12] using some basic connectives from propositional logic. These connectives are those used in our earlier discussion and are repeated below:

10.57 Logical connectives in meaning postulates
-> 'if... then'
* A 'and'
—1 'not'
v 'or'
= 'if and only if'

Let's look at some lexical relations in this approach, beginning with **hyponymy.** The hyponymy relationship between for example, *dog* and *animal* can be represented using —>, the *'if.. . then[1]* connective, by writing a rule like 10.58:

10.58 Vx(DOG(x) *ANIMAL(x))*

In the representation in 10.58 we use italic capitals to represent the translation of lexical items into predicate logic: 10.58 is to be read 'for all *x, if x is a* dog, then *x* is an animal', or more simply 'if something is a dog, then it is an animal.' In principle, all of the lexical relations described in chapter 3 can be represented using meaning postulates. We can look at a few further examples.

Binary antonyms

Here we can use the 'not' symbol (-.) as in 10.59 below:

10.59 *Vx(DEAD(x) -ALIVE(xf)*

This is to be read 'if something is dead then it is not alive.'

Converses

The lexical relation between the words *parent* and *child* can be captured as in 10.60:

10.60 a. Vx V>(B41?ENT(x, *y) -> CHILD^* x))
b. Vx *\/y(PARENT(x, y) -» ^CHILD(x> y))*

The formula in 10.60a tells us that if x is the parent ofjy then,jy is the child of x. The second forinula in 10.60b reflects the asymmetry of this relationship: if x is ys parent; x cannot be >'s child.

Synonymy

To capture the relation of synonymy we have to use two mirror-image z/. . . *then* rules, i.e. both of the rules in 10.61 below for a speaker for whom *couch* and *sofa* are synonyms:

10.61 a. V(COHCH(x) -> SOB4(x))
b. Vx(SOK4(x) -» *COUCH(xf)*

If both of these are true then *couch* and *sofa* are synonymous. We can abbreviate this double implication with the symbol = as in 10.62:

10.62 Vx *(COUCH&) = SOFAW)*

From these few examples we can see that this approach thus allows the formal semanticist to reflect the network of sense relations that we detect in the vocabulary of a language, in a format consistent with translation into predicate logic and interpretation via model theory.

These meaning postulates can be seen as a way of restricting of constraining denotation, e.g. 'if something is a dog, then it is an animal' tells us

something about the denotational behaviour of the word *dog.* If we take the view that the source for such information is the knowledge that speakers have, then we can see meaning postulates as an example of the effect of the subject's knowledge on the denotational properties of expressions.

The version of sentence and word meaning that we have outlined so far is only the starting point for a formal semantics of natural languages. The account has to be broadened to reflect the range of semantic features we find in all languages. In the next sections we discuss some of these developments.

10.7 Natural Language Quantifiers and Higher Order Logic

The theory of quantifiers that we have outlined so far suffers from several disadvantages as an account of quantifiers that are found in natural languages. One major problem, as we mentioned earlier, is that there are some common types of quantifiers which cannot be modelled in this standard form of the predicate calculus. We can briefly show why this is so by looking at the English quantifier *most.* It is impossible to establish *most* on a par with the universal quantifier V and existential quantifier 3, using the logical connectives A and —>.

Neither 10.63b or c below seem to have the same truth conditions as 10.63a:

10.63 a. Most students read a book.
b. Most x(5(x) A *R(x, hf)*
c. Most x(5(x) —> *R*(x_3 6))

The expression in b has the interpretation 'For most x, x is a student and x reads a book' which suggests the likeliest paraphrase in English 'Most things are students and read books', which is of course quite different from the meaning of 10.63a. The formula in c has the interpretation 'For most x, if x is a student then x reads a book' which suggests 'Most things are such that if they are students they read a book.' The problem here is that *most* is quantifying over all the individuals in the domain rather than over all students. We can show how this will cause a divergence from the meaning of 10.63a. First we may recall the truth table for the material implication —given in chapter 4. We can apply this to our expression as follows:

10.64 *S(x) R(x, b)* (S(x) —> *R(x, />))*

1	T	T	T
2	T	F	F
3	F	T	T
4	F	F	T

Next let us decide for argument's sake that *most* means more than 50 per cent of the individuals concerned. So whenever the expression (S(x) —> *R(x,* £)) is true of more than 50 per cent of the entities in the situation, the sentence in 10.63a will be true. However the truth table in 10.64 tells us that (S(x) -> *R(x,* 6)) is true in a number of situations, for example when the individual is not a student (i.e. S(x) is false) but does read (i.e. *R(x, b)* is true), as in line 4 of the table. Consequently we would predict that 10.63c is true in a number of situations that do not reflect the meaning of *Most students read a book,* for example if a majority of students do not read a book but they are outnumbered by the non-students who do read a book.

What seems to be going wrong here is that our form of interpretation has quantifiers ranging over all individuals in the relevant situation whereas in noun phrases like *most students* the quantifier in determiner position seems to have its range restricted by the type of thing named by the following noun.

A second problem with our predicate logic account of quantifiers also concerns the interpretation of noun phrases. In chapter 1 we discussed the compositionality of meaning and claimed that semantic rules need to parallel the compositionality and recursion that we find in grammar. However we can compare the following sentences and their translations into predicate logic:

10.65 a. [$_{NP}$Ray] [yp is hardworking] \
b. H(r)

10.66 a. [$_{NP}$ One student] [$_{VP}$ is hardworking]
b. (3x)XS(x) A H(x))

10.67 a. [$_{NP}$ All students] [$_{VP}$ are hardworking]
b. (Vx) (S(x) H(x))

In these examples the syntactic structure is the same: a noun phrase followed by a verb phrase. While in 10.65 the noun phrase corresponds to a unit in the logical form, i.e. *Ray = r,* in the following two examples the noun phrase does not correspond to a unitary expression in the logical formulae. In 10.67 for example the English noun phrase corresponds to no single logical expression. The meaning of *all students* is split: part of the meaning is to the left of the head noun *students* in the choice of the quantifier V, while part occurs to the right in the choice of the connective The NP *one student* is similarly divided between 3, *student* and the connective A. We can call this the problem of **isomorphism.**

Both of these problems can be solved by taking a different approach to the semantics of noun phrases, as described in the next sections.

10.7.1 Restricted quantifiers

One step is to express the restriction placed on quantifying determiners by their head nominals. This can be done by adopting a different notation: that

of **restricted quantification. A** sentence like *All students are hardworking* would be represented in the restricted format by 10.68a below, compared to the standard format in 10.68b:

10.68 a. (Vx: S(x)) H(x)
b. (Vx) (5(x) -» H(x))

Here the information from the rest of the noun phrase is placed into the quantifying expression as a restriction on the quantifier. Similarly *One student is hardworking* is represented in the restricted format by 10.69a below, again contrasting with the standard format in 10.69b:

10.69 a. (3x: S(x)) H(x)
b. (3x) (S(x) A H(x))

Restricted quantification helps solve the problem of isomorphism: it has the advantage that the logical expressions correspond more closely to natural language expressions. If we compare 10.68a and b above, for example, in a the English noun phrase *all students* has a translation into a unitary logical expression: (Vx: S(x)). *Most students* would be represented as (Most x: (S(x)); *few students* as (Few x: S(x)), etc.

We should note that in English some quantifiers can stand alone, e.g. *everything, everybody, everywhere.* These will have to be translated into complex expressions in predicate logic, as in 10.70 and 10.71 below:

10.70	*everything*	every thing	(Vx: T(x))
	everybody	every person	(Vx: P(x))
	everywhere	every location	(Vx: L(x))

10.71	Everything is either matter or energy:	(Vx: T(x)) (Af(x) v E(x))
	Barbara hates everyone:	(Vx: P(x)) *H(b,* x)
	Everywhere is dangerous:	(Vx: L(x)) D(x)

As with the universal quantifier, some English words seem to incorporate an existential quantifier, e.g. *something, someone, somewhere.* These will be expanded in the translation into predicate logic, as shown below:

10.72	*something*	some thing	(3x: T(x))
	someone	some person	(3x: P(x))
	somewhere	some location	(3x: L(x))

10.7.2 Generalized quantifiers

Though restricted quantification seems to have advantages for representing the syntax-semantics interface, we still need to develop a way to provide a semantic interpretation for noun phrase formulae like (Most *x:* (5(x)) *most students,* (Vx: S(x)) *all students,* etc. Some influential recent research on the formal semantics of noun phrase semantics has focused on an application of set theory from mathematical logic, called **generalized quantifier theory.** We can outline this approach, beginning with an example of a simple sentence like *John sang* from sections 10.5.1-2 earlier, where we used set membership to interpret it. We used 10.73 below to claim that this sentence is true if the subject is a member of the set identified by the predicate.

10.73 a. [S0)]$^{M'}$ = 1 iff [/]^{Afl} e [S]^{AI}>
b. The sentence *John sang* is true if and only if the extension of *John* is part of the set defined by *sang* in the model M_x

A different approach is to reverse this and evaluate the truth of *John sang* by checking whether singing is one of the properties that are true of John in the situation. In other words we look for singing to be among the set of things John did for the sentence to be true. To do this however we need to give a new predicate-argument structure to the sentence (10.74b below) and a new semantic rule (10.74c) to replace those in 10.40 earlier:

10.74 a. John sang.
b. John(sang)
c. [John (sang)]^{Ml} = 1 iff [sang]^{M*} G [John]^{Ml}

We can paraphrase^ 10.74c as *John sang* is true if and only the denotation of the verb phrase *sang* is part of the denotation of the name *John* in the model Mj.'To capture this procedure by a rule like 10.74c involves viewing John as a set of properties: a set of sets. For our model in sections 10.4-5 above this might include properties like 'is a Beatle', 'sang', 'played guitar', etc. The noun phrase *John* denotes this set of sets. This is a shift from the standard predicate logic analysis of the denotation of a noun phrase like *John* as an individual.

This translation of a noun phrase as a set of sets was proposed by Montague (1969) and developed by Barwise and Cooper (1981) as an application of the mathematical notion of generalized quantifiers. Since sets of sets and the formula in 10.74b are not part of the predicate logic we have been using so far, this constitutes an extension into a higher-order, or second-order logic.

In this approach the semantic interpretation of the sentence *Most students are hardworking* will interpret *most students* as a set of properties and the sentence will be judged true if the set *are hardworking* is an element of the set *most students.* The semantic rule for *most* can be given as follows:[13]

10.75 Most (A, B) = 1 iff 1A n B | > | A - B |

We can paraphrase this as *'Most A are B* is true if the cardinality of the set of things that are both A and B is greater than the cardinality of the set of things which are A but not B', or more succinctly 'if the members of both A and B outnumber the members of A that are not members of B'. This assumes our earlier definition of *most* as more than 50 per cent and therefore claims that *Most students are hardworking* is true if the number of students who are hardworking is greater than the number who aren't.

Other quantifiers can be given similar definitions in terms of relations between sets, for example:

10.76 All (A, B) = 1 iff A G B
All A are B is true if and if only set A is a subset of set B.

10.77 Some (A, B) = 1 iff A n B 0
Some A are B is true if and only if the set of things which are members of both A and B is not empty.

10.78 No (A, B) = 1 iff A n B = 0
No A are B is true if and only the set of things which are members of both A and B is empty.

10.79 Fewer than seven (A, B) = 1 iff | A n B | < 7
Fewer than seven As are B is true if and only if the cardinality of the set of things which are members of both A and B is less than 7.

This analysis of noun phrases as generalized quantifiers has stimulated a large literature investigating the formal properties of quantifiers in natural languages and has led researchers to propose solutions to a number of descriptive problems. We cannot do justice to this literature here but in the next two sections we will select examples to illustrate this field of inquiry. The reader is referred to Keenan (1996) for an overview.

10.7.3 The strong/weak distinction and existential *there* sentences

One descriptive problem, discussed by Milsark (1977) and subsequently by Barwise and Cooper (1981), de Jong (1987) and Keenan (1987), concerns the distribution of NPs in existential *there* sentences. Some examples are below:

a. There is/isn't a fox in the henhouse.
b. There are/aren't some foxes in the henhouse.
c. There are/aren't two foxes in the henhouse.
d. PThere is/isn't every fox in the henhouse.
e. ?There are/aren't most foxes in the henhouse.
f. ?There are/aren't both foxes in the henhouse.

These sentences are used to assert (or deny in negative versions) the existence of the noun phrase following fee.[14] As can be seen, some quantifying determiners, including *every, most* and fearfe, are anomalous in this construction. The explanation proposed by Milsark (1977) is that there are two classes of noun phrases, weak and strong, and that only weak NPs can occur in these sentences. Subsequent work has sought to characterize this distinction correctly. One proposal, from Keenan (1987) uses the format of generalized quantifiers to explain the difference in terms of **symmetry.** One group of quantifiers expresses asymmetrical relations, that is to say that the order of their set arguments is significant. We can take the example of *all* and *most.* The form *All A are B* is not equivalent to *All B are A,* so that *All my friends are cyclists* does not have the same meaning as *All cyclists are my friends.* Similarly *Most A are B* is not equivalent to *Most B are A,* so that *Most football players are male* does not mean the same as Afosf *males are football players.* We can schematize this pattern as below, where **det** is the quantifying determiner:

10.81 Asymmetrical quantifiers
det (A, B) #= **det** (B, A)

Another group expresses symmetrical relations. Here we can use *some* and *two* as examples. *Some A are B* is equivalent to *Some B are A,* so that *Some skiers 'are Sudanese* can describe the same situation as *Some Sudanese are skiers.* Similarly *Two Nobel prize winners are Welshmen* is equivalent to *Two Welshman are Noble prize winners.* These can be schematized as:

10.82 Symmetrical quantifiers
det (A, B) = **det** (B, A)

The asymmetrical class is also called **proportional** because they express a proportion of the restricting set identified by the nominal. So for example to interpret NPs like *most foxes., all foxes, few foxes* etc. we need access to the number of the relevant set of/axes. The symmetrical class are not proportional in this sense. If we say *two foxes* we don't need to know how many other foxes are in the set in order to interpret the noun phrase. This class is called, by distinction, **cardinal** quantifiers since they denote the cardinality of the intersection of the sets A and B, i.e. the intersection of *two* and/axes in our example. Some quantifiers have both a cardinal and proportional reading, for example *many* and *few.* Compare the sentences in 10.83:

10.83 a. There are many valuable stamps in this collection.
b. Many of the stamps in this collection are valuable.

The interpretation of *many* in a is cardinal in that the sentence means that the number of valuable stamps is high. The interpretation in b is proportional since *many* is here calculated relative to the collection. It is reasonable to use b but not a if the collection is in fact a small one.

The proposal is that the asymmetrical, proportional class are strong quantifiers and create strong NPs. These strong NPs form the class of items that cannot be used in existential *there* sentences. Symmetrical, cardinal quantifiers on the other hand form weak NPs and can be used in these sentences. The theory of generalized quantifiers allows us to characterize the difference in quantifiers reflected in the English data. One possible line of explanation for the difference is that the necessity in strong NPs for access to the restriction on the domain of quantification somehow clashes with the semantic function of existential *there* sentences. In other words, when interpreting *most foxes* we have to access the whole set of foxes, including those outside the set of *most.* The idea is that accessing a presupposed set of foxes clashes with the normal assertion or denial of the existence of foxes in sentences like 10.80a-c, creating a tautology or a contradiction respectively. See Barwise and Cooper (1981) for discussion.

10.7.4 Monotonicity and negative polarity items

A further descriptive problem that has been investigated in the generalized quantifier literature is how to account for the distribution of negative polarity items like English *any, ever, yet,* which seem dependent on the presence of negation in the sentence:

10.84 a. She doesn't ever eat dessert.
b. ?She ever eats dessert.

10.85 a. I haven't seen the movie yet.
b. ?I have seen the movie yet.

However as discussed in Laduslaw (1979, 1996) and van der Wouden (1997), the restriction seems to be wider than strictly sentence negation. As shown below, negative polarity items are also licensed by certain quantifiers like *nobody, few* and adverbials like *seldom, rarely,* as well as other items; see Laduslaw (1996) for more examples.

10.86 a. Nobody sees any difficulty.
b. ?Everybody sees any difficulty.

10.87 a. Few people have seen the movie yet.
b. ?Many people have seen the movie yet.

10.88 a. Rarely has she ever been late.
b. ?Often has she ever been late.

An influential proposal, deriving from Laduslaw (1979), is that the licensing expressions are not simply negative but have a particular property of

monotonicity. The term monotonicity applied to quantifiers describes patterns of entailment between sets and subsets. **Upward entailment** is characterized by entailment from a subset to a set. **Downward entailment** involves entailment from a set to a subset. Let's take as example (7VP) *is driving home* which is a subset of *(NP) is driving.* By placing different quantified nominals into the subject position we can test the monotonicity of the quantifiers:

10.89 *Everyone is driving* does not entail Everyone *is driving home.*
Everyone is driving home does entail *Everyone is driving.*
Therefore: *every* involves upward entailment.

10.90 *No-one is driving* does entail *No-one is driving home.*
No-one is driving home does not entail *No-one is driving.*
Therefore: *no* involves downward entailment.

10.91 *Someone is driving* does not entail *Someone is driving home.*
Someone is driving home does entail *Someone is driving.*
Therefore: *some* involves upward entailment.

10.92 *Few people are driving* does entail *Few people are driving home.*
' . *Few people are driving home* does not entail *Few people are driving.*
Therefore: *few* involves downward entailment.

Quantifiers which trigger upward entailment are described as monotone increasing while those involving downward entailment are described as monotone decreasing.

The specific explanatory proposal in this literature is that negative polarity items are licensed by downward entailing expressions. We can see even from our simple examples that this correctly predicts the following pattern:

10.93
a. Few people are ever driving home.
b. No-one is ever driving home.
c. ?Everyone is ever driving home.
d. ?Someone is ever driving home.

Our examples so for have been of sets and subsets identified by the right argument of the quantifier, corresponding to the VP arguments, for example *Few {people, driving)* and its subset *Few {people, driving home).* The same quantifiers may show the same or different entailment patterns in the sets and subsets in the left argument, corresponding to the NP, for example *Few {people, driving)* and it subset *Few {drunk people, driving).* The examples below show that *few* is downward entailing in the left argument as it is in the right (10.92 above) but that *every* is downward entailing in the left argument though it is upward entailing in the right (as in 10.89 above):

10.94 *Few people are driving* does entail *Few drunk people are driving.*
Few drunk people are driving does not entail *Few people are driving.*
Therefore: *few* involves downward entailment (left argument).

10.95 *Every person is driving* does entail *Every drunk person is driving.*
Every drunk person is driving does not entail *Every person is driving.*
Therefore: *every* involves downward entailment (left argument).

This difference correctly predicts that *every* licenses negative polarity items in the NP but not in the VP:

10.96
a. [Everyone who has ever driven drunk] will be ashamed by these figures.
b. ? [Everyone who has driven drunk] will ever be ashamed by these figures.

10.7.5 Section summary

In this section we have seen something of the formal investigation of quantifiers in natural language. We can identify two claims which emerge from this literature. The first is that formal models can be successfully developed to describe natural language quantifiers. The second, more ambitious, claim is that these formal models help identify and characterize features of quantifier behaviour that would otherwise remain mysterious.

10.8 Intensionality

10.8.1 Introduction

As we mentioned in the introduction, section 10.1, one disadvantage of the simple version of the denotational approach is that it downplays the speaker-hearer's **subjectivity.** The procedures we have been outlining allow a mechanical-seeming matching between statements and situations. However, as we have seen in our previous chapters, it is clear that natural languages largely communicate **interpretations** between speakers and hearers. For example languages contain a whole range of verbs which describe different mental states. Instead of a flat statement S, we can say in English for example:

10.97
a. Frank knows that **S.**
b. Frank believes that **S.**
c. Frank doubts that **S.**

d. Frank regrets that **S.**
e. Frank suspects that **S.**
f. Frank hopes that **S.**
g. Frank imagines that **S,** etc.

As we saw in chapter 5, one way of describing this, which comes to us from the philosophy of language, is to say that in sentences like 10.97 we have a range of speaker attitudes to the proposition expressed by S, or more briefly, that we have a set of **propositional attitudes.**

As we discussed in chapter 5, propositional attitudes are not only conveyed by embedding S under a higher verb. We might say that if a speaker chooses between the sentences in 10.98 below, the choice reflects a difference in propositional attitude between certainty and degrees of lack of certainty:

10.98 a. Phil misrepresented his income.
b. Phil probably misrepresented his income.
c. Phil may have misrepresented his income.

In another terminology, sentences which reveal this interpretative or cognitive behaviour are said to be **intensional** and the property is called **intensionality.** More generally these terms are applied whenever linguistic behaviour reveals a relation between an agent and a thought. The notion was discussed by Frege in his 1893 article 'Sense and Reference' *(fiber Sinn und Bedeutungi* see Frege 1980) in relation to cases where we need access to the sense of an expression as well as its denotation, as discussed in chapter 2. The classical cases are the verbs of propositional attitudes mentioned above, which in one terminology are said to form **opaque** contexts. The term opaque figuratively describes the fact that the truth or falsity of the subordinate clause seems to be independent of the truth or falsity of the whole sentences. As Quine (1980: 22-3) points out for the statements in 10.99:

10.99 a. Jones believes that Paris is in France.
b. Jones believes that Punakha is in Bhutan.

sentence 10.99a may be true and b false even though the components 'Paris is in France' and 'Punakha is in Bhutan' are true. Similarly for 10.100:

10.100 a. Jones believes that Punakha is in Nepal.
b. Jones believes that Paris is in Japan.

The sentence 10.100a may be true and b false even though the components 'Punakha is in Nepal' and 'Paris is in Japan' are both false. It's as if the subordinate clause (the belief context) is a walled-off, opaque domain, as

far as the truth-value of the main sentence is concerned. It seems that in such examples we need access to the content of the subject's belief, necessitating an extra level of sense, or in a more recent terminology, intension. The notion was developed formally by Richard Montague (1974); see Dowty (1979) for discussion.

The challenge for formal semantics is to develop the semantic model to reflect the interpretation and calculation that is so central to language. One strategy has been to enrich the formal devices in certain areas where intensionality seems most clearly exhibited in natural languages. Such areas include **modality, tense, aspect** and **verbs of propositional attitude.** In each of these areas there has been research into formal semantic accounts. We cannot go into these developments in any detail here, pausing merely to sketch some of the main areas of focus and to refer the reader to the relevant literature.

10.8.2 Modality

As we saw in chapter 5, modality is often described in terms of two related aspects of meaning. The first, **epistemic** modality, concerns the resources available to the speaker to express judgement of fact versus possibility. The second, **deontic** modality, allows the expression of obligation and permission, often in terms of morality and law. All languages allow speakers a range of positions in both of these aspects. If we take epistemic modality, for example, we can quote Allan's scale of modality in 10.101 below (1986: vol. 2, 289-90), which he views as a scale of implicatures such that each is stronger than the next about the fact of **p:**

a. I know that p.
b. **I** am absolutely certain that **p.**
c. I am almost certain that **p.**
d. I believe that **p.**
e. I am pretty certain that **p.**
f. I think that p.
g- I think/believe that **p** is probable.
h. **I** think/believe that perhaps **p.**
i. Possibly **p.**
j- I suppose it is possible that **p.**
k. It is not impossible that **p.**
1. It is not necessarily impossible that **p**
m. It is unlikely that **p.**
n. It is very unlikely that p.
o. It is almost impossible that **p.**
P- It is impossible that **p.**
q- It is not the case that **p.**
r. It is absolutely certain that not-p.

Even if we don't agree with Allan's selection or the ordering in this list, it is clear that there is a large range of options available to the speaker. Some of these choices of degree of commitment to the truth of p derive from the meaning of verbs like *believe, know,* etc.; others from negation; or from adjectives and adverbs like *possible* and *possibly.* The use of different intonation patterns can add further distinctions. In response to these facts about modality, **modal logics** were developed. The simplest approach employs a twofold division of epistemic modality into **fact** versus **possibility,** or 'situation as is' versus 'situation as may be'. One way of discussing this distinction between the actual and the non-actual is to talk of **possible worlds,** a phrase derived from Leibniz and formally developed by Kripke (see for example Kripke 1980). This is a difficult and controversial area in the philosophical literature but the notion has been important in formal semantics (see for example Lewis 1973, 1986). We can recognize the idea that a speaker, in moving away from certainty, can envisage two or more possible scenarios. So if we say *Fritz may be on the last train,* we entertain two situations: one where Fritz is on the train and another where he is not. Thus we imagine one situation where the statement *Fritz is on the last train* is true and another, where it is not. One way of dealing with this is to see truth as being relativized to possible situations, or possible worlds, to use this terminology.[15]

To reflect this, logicians introduce two logical operators 0 'it is possible that' and 'it is necessary that'. These can be put in front of any formula of the predicate logic, i.e.

10.102 0(f> = it is possible that 0
□0 = it is necessary that 0

The semantic definition of these relies on this new ontology of possible worlds: means 'true in all possible worlds' (i.e. no alternatives are envisaged by the speaker) and 0 means 'true in some possible worlds' (i.e. the speaker does envisage alternative scenarios). The formal implication of this is that truth must be relativized not to one situation but to one amongst a series of possible situations (worlds), including the actual situation (world). This means that our model must be expanded to include this multiplicity of situations, i.e. now *M - {W, U, F}* where as before *U* = the domain of individuals in a situation, *F* is the denotation assignment function, and the new element *W* is a set of possible worlds.

Relativizing truth to possible worlds enables one to adopt extensionally defined versions of Frege's notion of **sense** *(Sinn),* distinguished from **reference** *(Bedeutung),* as discussed in chapter 2. Using the term **intension** for sense, we can say that in this approach the intension of an expression is a function from possible worlds to its extension. In other words the function will give us the denotation of a particular linguistic expression in possible circumstances. Thus the intensions of nominals (NP), informally viewed as individual concepts, can now be viewed as functions from possible

worlds to individuals; the intensions of predicates (VP), characterized as properties, can be viewed as functions from possible worlds to sets of individuals; and the intensions of sentences (S), characterized by Frege as the thoughts expressed by sentences, i.e. propositions, can be viewed as functions from worlds to truth-values. See Chierchia and McConnell-Ginet (2000: 257-328) for discussion.

This approaches raises interesting issues: for example, how many possible situations are relevant to a specific utterance? How are the possible situations ranked, by a combination of the linguistic expressions and background knowledge, so that some are more probable than others? We cannot pursue these issues any further here; readers are referred to Allwood et al. (1977: 108-24), Cann (1993: 263-81) and Chierchia and McConnell-Ginet (2000: 257-328) for introductory discussions.

The second type of modality, deontic modality, has been treated in a similar way: as a projection from the world as it is to the world as it should be under some moral or legal code, i.e. as the speaker entertaining an idealized world. Deontic modal operators have been suggested for logic, including O0 'obligatorily that 0' and P0 'permitted that 0'.The former can be interpreted denotationally as 'true in all morally or legally ideal worlds' and the latter as 'true in some morally or legally ideal worlds'. Again see Allwood et al. (1977: 108-24) for discussion.

10.8.3 Tense and aspect

These two further important intensional categories are, as discussed in chapter 5, related to the speaker's view of time. We need not review our earlier discussion here, but in denotational terms, the speaker's ability to view propositions as timeless and eternal as in sentences like *All men are mortal,* or as fixed in relation to the time of utterance, or some other point identified in the metaphorical flow of time, clearly has truth-conditional implications. Take for example the sentences in 10.103 below:

10.103 a. The Irish punt will be replaced by the euro.
b. The Irish punt was replaced by the euro.

These sentences differ in truth-value being read by you today rather than say, in January 2002, and the only difference between them is their tense. We saw that an utterance can only be given a truth-value relative to a situation: it seems that part of the character of situations may be their location in time.

One response has been to incorporate time into model-theoretic semantics. One way to do this is to include tense operators, similar to the modal operators we have just mentioned. We might for example include three operators: **Past(0), Present(0)** and **Future(0).** This would allow formulae like 10.104 below:

Figure 10.1 Instants in the flow of time

i» i² i³ iⁿ

---------------- -------------------- -------------------- -------------------- ----------------- > TIME

10.104 a. **Past(C** (t, 7)) Tom chased Jerry.
b. **Present** (C (t, 7)) Tom is chasing Jerry.
c. **Future(C** (r, 7)) Tom will chase Jerry.
Key: *C:* chase
t: Tom
7: Jerry

Such tense operators rely upon a division of the flow of time into a series of ordered instants, as in 10.105 below, where **i** = instant and < = before:

10.105 $\mathbf{i^1 < i^2 < i^3 .. . < i^n}$

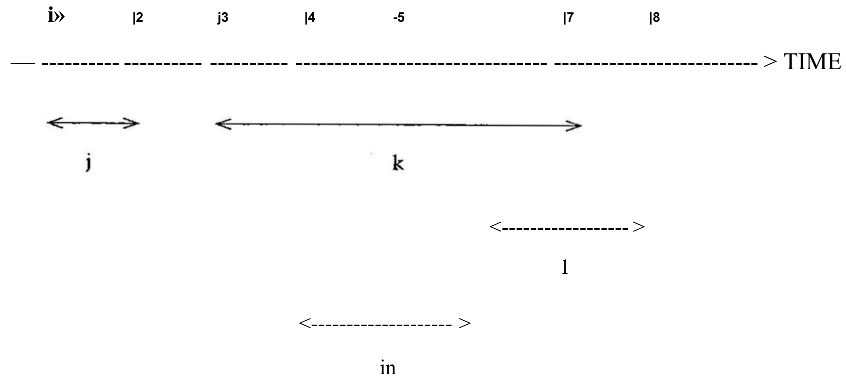
or alternatively as in figure 10.1. If we select instant i^3 in figure 10.1 as **now,** then the evaluation procedure for the formula **Past(C** (t, j)) in 10.102a above will state that it is true if *C* $(t_3\ j)$ is true at time $\mathbf{i^n}$, where $\mathbf{i^n < i^3}$; that is, if it is true at a time before now. In other words the model will relativize formulae to both a situation and a time, so that our model is now *M* = < IF, 17, F, 7, < >, where *I* are the instants in time and < is the ordering relation 'before'. See Gamut. (1991, vol 2: 32-44) and Cann (1993: 233-51) for introductory discussions of tense logics.

We saw in chapter 5 that tense is inextricably linked to **aspect,** a speaker's choice of viewing a situation as complete or incomplete, stretched over time or punctal, depending on the aspectual parameters of the language. When we come to consider the distribution of an activity or state over time, one useful modification to our simple model of time is to allow **intervals** of time in addition to just points or instants. Intervals can be defined in terms of instants: thus we can have an interval k which will be a continuous series of instants stretching between an initial and final instant, say $\mathbf{i^3}$ to $\mathbf{i^7}$. We can represent this as $\mathbf{k = [i^3, i^7]}$. Intervals can be ordered with respect to other intervals in various ways, some of which we can show in diagram form in figure 10.2. Here interval **j** precedes interval **k;** interval **1** overlaps **k;** and **m** is contained within k. This treatment of intervals might allow description of stretches of time, and interrelations between times, like those in 10.106:

10.106 a. I studied Hausa for three years, then gave it up.
b. She was ill all last week, when the interviews took place.

Formal approaches have to cope with the various aspectual and situation type distinctions we looked at in chapter 5. Cann (1993: 25 Iff.) proposes, for example, a perfective aspect operator **Perf** and an imperfective operator

Figure 10.2 Intervals of time



Impf for predicate logic, which will further relativize the truth of logical formulae. These operators rely on the idea of intervals of time. Without giving the formal definitions, a perfective formula will be true if both the start and end instants are included before the reference time point,[16] thus reflecting the complete interpretation of the perfective aspect. An imperfective formula on the other hand will be true if the activity overflows the time interval that is being interpreted. Thus our sentence 10.104a above, repeated as 10.107a below, can be given the simple perfective interpretation as in 10.107b:

10.107 a. Tom chased Jerry.
b. **Past(Perf(C** (r, 7)))

The evaluation procedures for this formula will state that it is only true if the action of chasing is complete before the time of utterance. We can compare this with the imperfective clause in 10.108a below, represented in the formula in 10.108b:

10.108 a. Tom was chasing Jerry (when I opened the door).
b. **Past (Impf** (C (t, 7')))

Here the evaluation procedure will require that for 10.108b to be true the time interval for the chasing activity (C) should overlap the door opening event.

These are of course only preliminary sketches of the task facing formal semanticists: to model formally the tense and aspect distinctions found in languages, some of which we saw in chapter 5. See Cann (1993: 251-62) for further discussion.

In the next section we discuss attempts to model formally the dynamism and context-dependence of language use.

10.9 Dynamic Approaches to Discourse

Our discussion of formal semantics so far has been concerned with the analysis of individual sentences. However, as we discussed in chapter 7, sentences are uttered in a context of discourse and many features of language reveal speakers' efforts to package their messages against the current context, in particular to take account of their hearers' knowledge and interpretive task. There have been a number of proposals to model formally the influence of discourse context on meaning, including File Change Semantics (Heim 1983, 1989) which uses the metaphor of files for information states in discourse and Dynamic Semantics (Groenendijk and Stokhof 1991, Groenendijk, Stokhof and Veltman 1996), where meaning is viewed as the potential to change information states. In this section we focus on one further approach, Discourse Representation Theory (DRT) (Kamp and Reyle 1993) and look briefly at how it attempts to model context dependency. From a wide range of issues discussed in this theory we shall select just one: discourse anaphora. Our account will be an informal one; the technical details can be found in Kamp and Reyle (1993). We begin by sketching in the background.

10.9.1 Anaphora in and across sentences

In chapter 7 we discussed the anaphoric use of pronouns. Traditionally the pronoun *himself* in-1.0.109 below is said to gain its denotation indirectly through coreference with the preceding nominal, *James.* They are said to be coreferential, i.e. denote the same entity in the situation described. As shown in a below, this can be reflected by attaching referential subscript indices; and as b shows, in predicate logic this relationship can be represented by giving each nominal the same individual constant:

10.109 a. James, mistrusts himself,.
b. M(j, j)

Since quantified nominals don't directly denote an individual, sentences like 10.110a below are given a representation like 10.110b in predicate logic, where the pronoun is treated as a variable bound by a quantifier:

10.110 a. Every thief mistrusts himself.
b. (Vx: T(x)) *M(x, x)*

We also discussed in chapter 7 how new entities are often introduced into a discourse by an indefinite noun phrase and thereafter referred to by a range of definite nominals varying in their informational status, including pronouns. Once again, in an example like 10.111 below the pronoun is said to be anaphorically related to the preceding indefinite NP:

10.111 Joan bought a carj and it, doesn't start.

In predicate logic this use of indefinite nominals can be treated as a kind of existential assertion and the pronoun again treated as a variable bound by the quantifying expression, as shown below:

10.112 (3x: C(x)) B(j, x) A _.S(X)
Paraphrase: There is car such that Joan bought it and it doesn't start.

This parallel between indefinite NPs and quantifiers breaks down in cross-sentential anaphora. For quantifiers the representation correctly predicts that anaphoric pronouns cannot occur outside the scope of the quantifier, such as in a following sentence. See the example below, where we assume the two sentences are spoken in sequence by the same speaker:

10.113 a. Every girl, came to the dance.
(Vx: G(x)) G(X, d)
b. ?Shej met Alexander.
Af(x, *a)*

In the logical form in b the variable x is not bound by the quantifier in the preceding sentence and is therefore uninterpretable. This correctly predicts the fact that the pronoun *she* in b cannot refer back to *every girl* in a.

However, indefinite NPs do allow cross-sentential anaphora; see for example:

10.114 a. A girlj came to the dance. She, met Alexander.

One way of reflecting this behaviour of indefinite nominals is to recognize that a discourse has a level of structure above the individual sentences and to view the role of indefinite nominals as introducing entities into this discourse structure. These are called **discourse referents** (Karttunen 1976) and the idea is that they have a lifespan in the discourse during which they can be referred to by pronouns. This lifespan can be limited by semantic operators such as negation. For example a discourse referent set up by an indefinite NP under negation has its lifespan limited to the scope of that negation. See the following example, where we assume the a and b sentences are uttered in succession by the same speaker:

10.115 a. Joan can't [afford a FerrariJ.
b. ?She likes itj though.

Here the pronoun *it* cannot refer back to the indefinite NP *a Ferrari* because the latter's lifespan as a discourse referent is limited by the scope of *not,* shown by square brackets. As we shall see, Discourse Representation Theory is one way of formalizing such a notion of discourse referents.

10.9.2 Donkey sentences

Even within a single sentence there are examples where anaphora between indefinite NPs and pronouns causes problems for a quantifier-variable binding account. If we take sentence 10.116a below we can represent it in standard predicate logic as 10.116b:

10.116 a. If Joan owns a Ferrari she is rich.
b. (3x (F(x) A O($_7$; x))) -> F(j)

However applying the same translation procedure to 10.117a gives us 10.117b:

10.117 a. If a teenager owns a Ferrari he races it.
b. (3x3y (T(x) A *F(y) A O(X,* y))) F(x, *y)*

Though these two sentences seem to have the same syntactic structure, 117b is not a legal formula because the variables in the consequent of the implication are not correctly bound by the relevant existential quantifiers. To capture the meaning of 10.117a in a well-formed formula we have to use something like 10.118 below:

10.118 a. VxVy ((T(x) A F(y) A O(x, y))) -» F(x, *y)*
b. Paraphrase: For all x, all *y:* if x is a teenager, *y* is a Ferrari, and x owns *y,* then x races *y*

This does capture the fact that the preferred interpretation of 10.117 has universal force, i.e. that all teenagers who have Ferraris race them. However the problem here is that we have translated the indefinite nominal *a Ferrari* by a universal quantifier expression in 10.118a and by an existential quantifier expression in 10.117b. This is a threat to the notion of compositionality and is another version of our isomorphism problem earlier. It seems unsatisfactory that an indefinite NP is sometimes treated as an existential quantifier and at other times as a universal quantifier, the deciding factor apparently being the presence of an anaphoric pronoun.

Examples like 10.117a are known as **donkey sentences** after Geach's (1962) discussion of this problem using examples like *If a farmer owns a donkey, he beats it* and *Every farmer who owns a donkey beats it.* In essence, the problem with the pronoun *it* in these examples is that it cannot be a referring expression, since there is no specific donkey it denotes. However, as we have seen, treating *it* as a bound variable leads to other problems.[17]

10.9.3 DRT and discourse anaphora

Discourse Representation Theory (DRT) formalizes a level of discourse structure which is updated by successive sentences and forms a representation

of the discourse referents introduced so far. The discourse referents form an intermediate level between the nominals and the real individuals in the situation described. The main form of representation is a Discourse Representation Structure (DRS), usually presented in a box format, as shown below. These DRSs are built up by construction rules from the linguistic input, sentence by sentence If we take the sentences in 10.119 below as uttered in sequence, the first sentence will trigger the construction of the DRS in 10.120:

10.119 a. Alexander met a girl$_£$.
b. She, smiled.

10.120
x y

Alexander (x)
Girl (y)
met (x, y)

The discourse referents are given in the top line of the DRS, called the universe of the DRS, and below them are conditions giving the properties of the discourse referents. These conditions govern whether the DRS can be embedded into the model of the current state of the discourse. A DRS is true if all of the discourse referents can be mapped to individuals in the situation described in such a way that the conditions are met. A name like *Alexander* in 10.119 denotes an individual, while an indefinite NP like *a girl* will be satisfied by any individual meeting the property of being a girl. The third condition is the relation *met (x, y).* We can see that the truth conditions for sentence 10.119a are given here by a combination of the discourse referents and the conditions. The sentence will be true of a situation if it contains two individuals; one named Alexander, the other a girl, and if the first met the second. An important point here is that in an example like this the introduction of a discourse referent into a DRS carries an existential commitment. Thus the indefinite NP *a girl* is treated as having existential force, though there are other ways of introducing indefinite nominals which do not have this existential commitment, as we shall see below. The initial DRS is labelled K^, the next K, and so on. The latest I< acts as the context against which a new sentence in the discourse is interpreted.

The second sentence in 10.119 updates the discourse and adds another discourse referent, *she.* The embedding rule for pronouns will say that we must find an accessible antecedent for it. In this case gender is a factor since *she* must find a feminine antecedent. If the correct antecedent for the pronoun is identified, the result is the extended version below of the original DRS with a new reference marker and a new condition:

10.121

x y u

Alexander (x)
Girl (y)
met (x, y)
u = y
smiled (u)

A negative sentence like 10.122 below will be assigned the DRS in 10.123:

10.122 Joan does not own a Ferrari.

x
Joan (x)

y
—> Ferrari (y)
owns (x, y)

Here the DRS contains one discourse referent and two conditions: the first is the usual naming relation, Joan (x); and the second is a second DRS embedded in the first and marked by the logical negation sign —The satisfaction of this second condition is that there is not a Ferrari such that Joan owns it. This contained DRS is said to be **subordinate** to the containing DRS and is triggered by the construction rules for negation. This subordination has two effects on any discourse referents within the embedded DRS. The first, as suggested by our characterization of how the condition in 10.123 is satisfied, is that there is no existential interpretation for discourse referents in this type of subordinate DRS. Thus there is no existential commitment with the indefinite NP *a Ferrari* in this sentence, unlike *a girl* in 10.119.

The second effect follows from the existence of **accessibility rules** in DRT. Briefly, proper nouns (names) are always accessible in the subsequent discourse, i.e. once introduced can always be referred to by an anaphoric pronoun. The accessibility of other nominals depends on the structure of the DRSs they occur in. For negatives, the rule is that discourse referents introduced within a subordinate DRS under the scope of negation are inaccessible to pronouns in subsequent stages of the DRS.[18] This means that the discourse referent *y* (i.e. *a Ferrari)* in 10.123 is inaccessible to subsequent pronouns. We can look at our earlier example 10.115, repeated below, to show this.

10.124 a. Joan can't afford a Ferrarij.
b. ?She likes itj though.

We can suggest 10.125 below as a DRS after the two sentences in 10.124:

10.125

x z
Joan (x)

y

Ferrari (y)
afford (x, y)

z - x
u = ?
likes (z, u)

The pronoun *she* in the second sentence is successfully interpreted as anaphoric with *Joan* in the first sentence, and hence ar = x in the DRS conditions. However we have written a question mark in the identification of an antecedent for *u* (i.e. *it)* because the only possible antecedent for *y* (i.e. *a Ferrari)* is not accessible since it occurs in the subordinate DRS box under negation. This explains the semantic anomaly of 10.124 above and provides a formalization of one aspect of the notion of discourse referent lifespan mentioned in section 10.9.1.

Sentences with conditionals are also represented with subordinate DRSs as conditions. The construction rules for these embed two DRSs linked by a connector =>, which parallels the material implication in predicate logic. The first DRS represents the antecedent and the second the consequent. Our earlier example *If Joan owns a Ferrari she is rich* would be given the complex DRS below (assuming an integration into a preceding empty DRS):

10.126

y
Joan (x)
Ferrari (y)
owns (x, y)

u

U = X
rich (u)

In this DRS the accessibility rule for names (that they are accessible to the whole of the subsequent discourse or have an 'eternal' lifespan, so to speak) is reflected by the discourse referent *x* (for *Joan)* being represented in the containing DRS, outside the subordinate boxes for the antecedent and consequent.

A donkey sentence like 10.117 earlier *If a teenager owns a Ferrari he races it* would be given a DRS like the following:

10.127

x y	u v
Teenager (x) Ferrari (y) owns (x, y)	u = x v = y races (u, v)

The accessibility rules for conditional sentences state that the antecedent discourse referents are accessible from the consequent but not vice versa, i.e. anaphora can reach 'up and back' but not 'down and forward'. This means that the pronoun *it* can refer anaphorically to *a Ferrari* in 10.117 because the discourse referent in the antecedent is accessible to the pronoun in the consequent. On the other hand a sentence like *?If a teenage^ owns it$_k$ hei races a Ferrari$_k$* is anomalous because the indefinite nominal in the consequent is not accessible to *it* in the antecedent.

Sentences with universal quantifiers are given a representation like conditionals; 10.128 below can be given the DRS in 10.129:

10.128 Every teenager who owns a Ferrari is rich.

10.129,.

x y	
Teenager (x) Ferrari (y) owns (x,'y)	rich (x)

A donkey sentence with *every* like 10.130 below is therefore given the DRS 10.131, which we can compare with 10.127 above:

10.130 Every teenager who owns a Ferrari races it.

10.131

x y	u
Teenager (x) Ferrari (y) Owns (x, y)	u = y races (x, u)

This representation brings together the two forms of donkey sentences into a structurally similar representation.

All of these conditional DRSs share an accessibility rule: any discourse referent introduced in a subordinate DRS is inaccessible to pronouns in a condition outside the subordinate DRS.This explains the impossible anaphora in 10.132 below, which would have the DRS 10.133:

10.132 Every student reads [a book on semantics]$_?$?Itj is heavy.

10.133

<table>
<tr><td>X

student (x)</td><td>=></td><td>y

Book on semantics (y)
Read (x, y)</td></tr>
<tr><td colspan="3">u = ?
heavy (u)</td></tr>
</table>

In 10.133 we again use a question mark to show that the pronoun *it* in the second sentence cannot be anaphorically related to any antecedent nominal. The discourse referent *book on semantics* is not accessible to the pronoun because it is in a subordinate DRS while the pronoun is in the superordinate DRS. This accessibility constraint explains the difference between indefinite nominals and quantified nominals in licensing a subsequent pronoun. Compare 10.132 above with 10.134 below:

10.134 A student read [a book on semantics]$_?$ Itj was heavy.

In 10.134 the pronoun can be anaphorically related to the indefinite NP *a book on semantics* because the structure of the DRS involves no subordination.

We leave our brief review of DRT at this point. Our discussion has revealed that the theory has a number of advantages in the description of discourse anaphora. The theory formalizes the notion of discourse referents and provides a unified explanation for the lifespan in the discourse of different nominals. In particular we saw that DRT distinguishes between names, which are always available for subsequent anaphoric pronouns, and indefinite NPs, whose lifespan depends on the type of sentence they occur in, for example: positive assertions, negative sentences, conditional sentences, and universally quantified sentences. The theory brings out the similarity between conditional and universally quantified donkey sentences and collapses the treatment of indefinite nominals in donkey sentences to the general cases. Finally DRT's view of an incrementally adjusted discourse structure seems very appealing in the light of our discussion in chapter 7. This structure can be viewed as one facet of the kind of knowledge representation that we described in chapter 7 as being cooperatively managed by participants in discourse.

10.10 Summary

In this chapter we have attempted an outline of how a formal semantic analysis might proceed. We have looked at how English sentences might be translated into a logical metalanguage, the predicate logic, and how this logic can be given a denotational semantics via model theory. We began with the translation and interpretation of simple statements. We then looked at quantification by discussing sentences with the universal quantifier V and the existential quantifier 3, and looked at compound sentences, using the example of the connective A 'and'. We then turned briefly to word meaning in this approach. Having sketched in this basic formal model, we began to look at how it has been extended to reflect important features of natural language semantics. We began by looking at how the treatment of quantifiers in first-order predicate logic has to be extended to reflect natural language quantifiers. We saw how the notion of generalized quantifiers has been applied to solve descriptive problems in English quantifiers. We turned then to the treatment of pronominal anaphora and looked at how Discourse Representation Theory models anaphora within and between sentences by establishing a dynamic model of discourse structure.

In the simple model we concentrated on the extensions of expressions like nominals, predicates and sentences. We have seen, however, that in a number of different ways we need to expand such a model to take account of intensional features of language. The developments we have touched on - possible worlds, models of time and aspect - are mechanisms introduced to reflect this intensionality. It is at this point - intensions - that we can perhaps see denotational approaches coming into contact with representational approaches. For the latter will ask the essentially psychological question about intensions: how is it that speakers identify a relationship between a word and its extension? If we look back to our model M_1 *we* can see that we used a function F_x to return the denotations of constants and predicates in the situation. It is this function, relating the logical translation of nouns like *cat* and *dog* to the entities in the situation, for which representational approaches will seek a psychological explanation. It might thus be possible to view the different traditions of denotational and representational semantics as complementary lines of enquiry, concerning themselves with two related aspects of meaning.

FURTHER READING

There are several very good introductions to logic and the choice depends on the reader's taste. Allwood et al. (1977), and McCawley (1981) are intended for a linguistics audience. Other more general introductions are Guttenplan (1986) and McKay (1989).

There are a number of good introductions to formal semantics: Chierchia and McConnnell-Ginet (2000) and Cann (1993) both provide in-depth descriptions of the kind of model-theoretic semantics outlined in this chapter. De Swart (1998) and Portner (2005) provide concise and accessible introductions, while Bach (1989) is an engaging and non-technical introduction in lecture format. Lappin (1996) is a comprehensive collection of papers which review topics of contemporary research

in formal semantics. Portner and Partee (2002) present an important selection of primary articles. Gamut (1991) consists of two volumes: the first is an introduction to logic; the second deals with intensional logics and formal semantics, and includes an introduction to Montague Grammar. The basic reference for Discourse Representation Theory is Kamp and Reyle (1993).

EXERCISES

10.1 Translate the following sentences into predicate logic. For compound sentences use the truth-functional connectives we employed in this chapter. Some symbols are provided for your use:

[Symbols: *a* = Arthur, w = Merlin, *g* = Guinevere, Z = Lancelot, *e* = the sword Excalibur, K"(x) = *x* was a king, Q(x) - *x* was a queen, lF(x) = *x* was a wizard, z4(x, *y)* =*x* advised y, P(x, *y)* = *x* possessed *y, L(x, y)* = *x* loved 3/]

a. Arthur was a king and Merlin was a wizard.
b. If Arthur was a king, then Guinevere was a queen.
c. Arthur, who was a king, possessed the sword Excalibur.
d. Merlin did not advise Lancelot.
e. Either Lancelot loved Guinevere or Guinevere loved Lancelot.
f. Merlin was a wizard who advised Arthur.

10.2 Translate the following sentences into predicate logic, using the restricted format for 3 and V, the existential and universal quantifiers, as necessary. Note which sentences, if any, allow two interpretations.

I [Symbols: Z = Lancelot, *h* = the Holy Grail, D(x) = x is a dragon, N(x, *y)* - x was nervous of y, K(x, *y)* = x was keen on y, H(x, *y)* - *x* hated y, S(x, y) = x searched for y]

a. Lancelot hated all dragons.
b. Every dragon was nervous of Lancelot.
c. One dragon was nervous of everyone.
d. Someone searched for the Holy Grail.
e. Every dragon wasn't keen on maidens.
f. Every dragon who was keen on maidens was nervous of Lancelot..
g. Not everyone searched for the Holy Grail.
h. No dragon searched for Lancelot.

10.3 For the following exercise, assume the model Af_3 specified below:

U_3 - {Lancelot, Gawaine, Elaine, Igraine, dragon}
F_3(Z) = Lancelot, F_3(^) = Gawaine, $F_3(e)$ = Elaine, $F_3(i)$ = Igraine, F_3(d) = dragon
$F_3(M)$ = was a maiden = {<Elaine>, <Igraine>}
F_3(K) = was a knight = {<Lancelot>, <Gawaine>}
F_3(D)= was a dragon = {<dragon>}
F_3(L) = loved = {eElaine, Lancelots cigraine, Gawaine>, <Gawaine, Igraine>}
F_3(C) = captured = {<dragon, Elaine>, <dragon, Igraine>}
F_3(S) = slew = {<Lancelot, dragon>}.
F_3(F) = freed = {<Lancelot, Elaine>, <Lancelot, Jgraine>}

Calculate the truth-value of the following sentences with respect to $A4_3$:

a. *L(g>I)*
b. C(d, Z)
c. (Vx: A4(x)) L(x, *g)*
d. (3x: Af(x)) L(x, ^)
e. *S(l> d)* A -i(3x: K(x)) L(x, *e*
f. (Vx: D(x)) *S(l, x)* A *(Vy. MCyY) F(l, y)*

10.4 Assuming the' truth tables for the connectives given in 1 a-c below, evaluate the truth of the sentences in 2 with respect to the same moaef JVf_3 above:

1 a.

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

b.

p	q	p v_e q
T	T	F
T	F	T
F	T	T
F	F	F

c.

p	q	p -? q
T		T
T	F	F
F	T	T
F	F	T

2 a. F(Z, *e)* v F(Z, *i)*
b *F(l, e)* v_t *F(l> i)*
c. *S(l₃ d) F(l> e)*
d. *U& i) -> ?(g, i)*

10.5 Use the formulae of meaning postulates to represent the semantic relations between the following pairs of words:

sweater/jumper
true/false
gun/weapon
open/shut (of a door)
uppercut/punch (in boxing)
car/automobile

10.6 In section 10.8.1 we used verbs of propositional attitude like *believe* to show the phenomenon of opaque contexts. Try to come up with four or five sentences where the truth or falsity of the subordinate clause seems to be independent of the truth or falsity of the whole sentences.

In section 10.7.3 we discussed the symmetry of quantifiers. For each of the quantifiers below decide whether it is symmetrical of asymmetrical:

a. many (in its cardinal use)
b. few (in its cardinal use)
c. every
d. (at least) four

10.8 In section 10.7.4 we discussed the monotonicity of quantifiers. For each of the quantifiers below use your own examples to decide whether they are (a) upward or downward entailing in the left argument and (b) upward or downward entailing in the right argument:

a. most
b. many
c. (exactly) two-

10.9 Below is a mini-discourse of two sentences. Assume that there is no preceding context. Give a DRT Discourse Representatidh Structure (DRS) for the first sentence and a second updated DRS after the second is embedded:

A man bought a donkey. He fed it.

10.10 Using the DRT accessibility rules discussed in the chapter, try to identify which NPs in the following sentences introduce discourse referents that are accessible for coreference with pronouns in subsequent sentences.

a. If Carl drinks a beer he is happy.
b. Maura does not own a scanner.
c. Every student who does an exercise enjoys it.

NOTES

1 This term describes the studies in formal semantics which have followed the work of Richard Montague. As mentioned in chapter *4,* Montague hypothesized that the methods of logic could be used to analyse the semantics of English and other natural languages: ‘There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians: indeed, I consider it possible to comprehend the syntax and semantics of both kinds of language within a single, natural and mathematically precise theory’ (Montague 1974: 222, cited in Cann 1993: 2)? For introductions to Montague semantics see Dowty et al. (1981) and Cann (1993).

2 This quotation refers to **possible states of affairs.** This another way of referring to the notion of **possible worlds.** We met this notion briefly in chapter 5 when we discussed modality; we come back to it again later in this chapter.

3 See Haack (1978)*-for an accessible description of the development of modern logic, and its philosophical background.

4 For an introductory discussion of animal communication systems, see Akmajian, Demers and Harnish (1984: 9-45).

5 For a modern translation of Frege’s work into English, see Frege (1980). Frege’s work on the logic of quantifiers seems to have been independently paralleled in the investigations of the logician Charles Sanders Peirce. See Haack (1978: 39ff.).

6 For example, we shall not deal with the fundamental relationship between syntactic rules and semantic rules that is characteristic of Montague Grammar.

7 Note that though the universal quantifier sets up a range of applicability for the generalization, it does not carry any existential commitment. Our expression in 10.23 is equivalent to saying that if there were students, then they wrote a paper (or more opaquely perhaps, there is no such thing as a student who didn’t write a paper). Because of the truth behaviour of material implication, discussed in chapter 4, if there are no students, then our sentence is vacuously true. So, rather counter-intuitively, *Every student wrote a paper* is held to be true when there are no students. We can show this with the following truth table (based on the table for described in 4.28 in chapter 4):

Sx lP(x, p) *(Sx* IT(x, />))

1	T	T	T
2	T	F	F
3	F	T	T
4	F	F	T

If there are no students (no thing is a student) then lines 3 and 4 of this table apply and in both the whole expression is true. Clearly though, it would be very odd to say *Every student wrote a paper* when there were no students and therefore no papers. One way of explaining this is to say that it is because the universal quantifier is quantifying over the universe of individuals, whether they are students or not. In section 10.7.1 below we discuss proposals to restrict the range of the quantifier to the type of things named by the nominal, here students. The existential quantifier 3 described below in the text does carry an existential commitment.

8 As we noted earlier, some predicates only require one argument, e.g. *Fred smokes* •$(/), others two, *Pat resembles Beethoven R(p, b),* or three *Giovanni gave the cello to Mike G(g,* c_3 *m).* In logic any number of arguments is theoretically possible; in English, of course, the normal requirements for a verb would be one, two, or three arguments (with a few verbs like *bet* having four).

9 We will assume the following set theory notion and representations:

1 A **set** {..}, which can be identified by listing the members, e.g. {Mercury, Mars, Earth, . . . } or by describing an attribute of the members, e.g. {x: x is a planet in the solar system}.
2 **Set membership, x** e **A,** e.g. Mercury G {X: x is a planet in the solar system}.
3 **Subset, A c** B, where every member of A is a member of B, e.g. {Venus, Jupiter} c {x: x is a planet in the solar system}.
4 **Intersection** of sets, **A** n **B,** which is the set consisting of the elements which are members of both **A** and **B,** e.g. {Venus, Mars, Jupiter, Saturn} n {Mars, Jupiter, Uranus, Pluto} = {Mars, Jupiter}.
5 **Ordered pair,** <*a, b*>$_3$ where the ordering is significant, e.g. <Mercury, Venus> * <Venus, Mercury>.
6 **Ordered n-tuple,** <a_{15} a_2, a_3.. . *a,,*>$_3$ e.g. the 4-tuple <Mercury, Venus, Earth, Mars>.
7 **Cardinality** of A, | A |, which is the number of members in A.
8 | **A** | = *five,* the cardinality of **A** is five, i.e. **A** has five members.
9 | **A** | > | **B** |, the cardinality of **A** is greater than **B;** i.e. **A** has more members than B.
10 | **A** | > | **B** |, the cardinality of **A** is greater than or equal to **B;** i.e. **A** has the same or more members than **B.**
11 **A - B, A** minus **B,** the set of members of A that are not also members of B.
12 **0** is the empty set.

10 We ignore here the logic possibility but murine improbability that Jerry hunts himself.

11 Of course, in our informal presentation here we necessarily take on trust these advantages of formality and explicitness: we have not investigated the formal nature of sets, functions, relations and the logics. For an excellent introduction to the mathematical foundations of these notions, see Partee, ter Meulen and Wall (1990).

12 Note that since meaning postulates express relationships between the **extensions** of linguistic expressions, they constitute knowledge about the world rather than about words.

13 This formulation is described as the **relational** view of quantifying determiners since it treats the determiner as a two-place predicate taking sets as arguments,

i.e. as denoting a relation between sets. An alternative is the **functional** view where the determiner is a function that maps a common noun denotation onto a noun phrase, which is the generalized quantifier. The generalized quantifier then takes a VP denotation as an argument to build propositions . See Keenan (1996) for discussion and Chierchia and McConnell -Ginet (2000: 50Iff.) for an introductory description.

14 This existential *there* construction must be distinguished from other sentences beginning with *there,* for example the use of *there* to introduce lists, as in A: *Which paintings do you have left?* B: *Well, there's the Picasso, the Rembrandt, and the Klee.* This construction behaves differently, allowing for example: *There's most of the Impressionists, and there's both Kandinskys.*

15 For a discussion of the application of possible world semantics to the issue of fictional entities and worlds that we discussed in chapter 2, see Lewis (1978).

16 As we saw in chapter 5, the reference time point may be the time of utterance as in the perfective in 1 below; or a time in the future or past of the time of utterance, as in the perfectives in 2 and 3:

1 He has served three presidents.
2 By next year, he will have served three presidents.
3 By 1992, he had served three presidents.

17 Seuren (1994: 1060) points out another problem for a bound variable analysis: that is that our translation via the universal quantifier V in 10.118 lacks gen - erality because a similar scope problem occurs in sentences like *If it's a good thing that Smith owns a donkey , it's a bad thing that he beats it* and *Either Smith no longer owns a donkey or he still beats it.* For discussion of donkey sentences see Kamp (1981),- Reinhart (1986), Heim (1990) and Seuren (1994).

18 For a discussion of counterexamples to this generalization, and a proposal for a solution, see Krahmer (1998: 65ff.).